

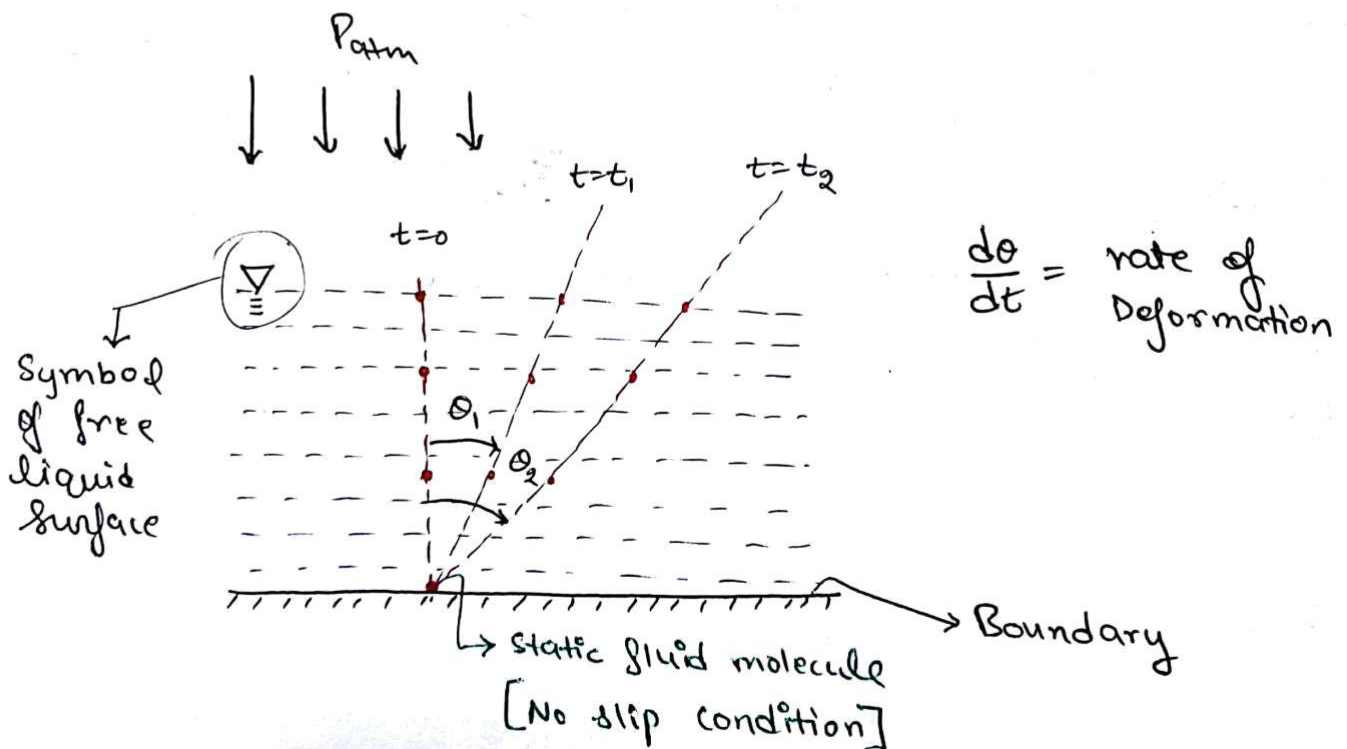
# FLUID MECHANICS

## CHAPTER - 1

# INTRODUCTION & FLUID PROPERTIES

- Fluids are the substances that are having the ability to flow or deform continuously under the action of shear force
- For a static fluid shear force is zero
- examples of fluid
  - water
  - mercury
  - Air
  - steam
  - etc

Note: All types of liquids and gases are known as fluids



## Difference b/w solid & fluids

- In case of solids on application of load the deformation is constant w.r.t. time whereas in case of fluids on application of force, the deformation is continuous and hence in solids deformation is important whereas in fluids  $\frac{d\theta}{dt}$  [rate of deformation] is more important.
- In case of solids on removal of loads the solids will try to regain their original position whereas in case of fluids they will never try to regain their original position on removal of load.
- Solids will show resistance to all types of loads whereas fluid will only show resistance to compressive loads.
- In case of solids the deformation can be linear as well as angular whereas in case of fluids deformation is only angular.

Note: Substances are differentiated into solids or fluids on the basis of their behaviour under load.

### Cohesion

it is the intermolecular forces of attraction b/w molecules of same nature.

### Adhesion

it is the intermolecular forces of attraction b/w molecules of different nature

ex:- water on glass will show adhesion more whereas mercury on glass will show cohesion more, water on plastic sheet will show cohesion more.

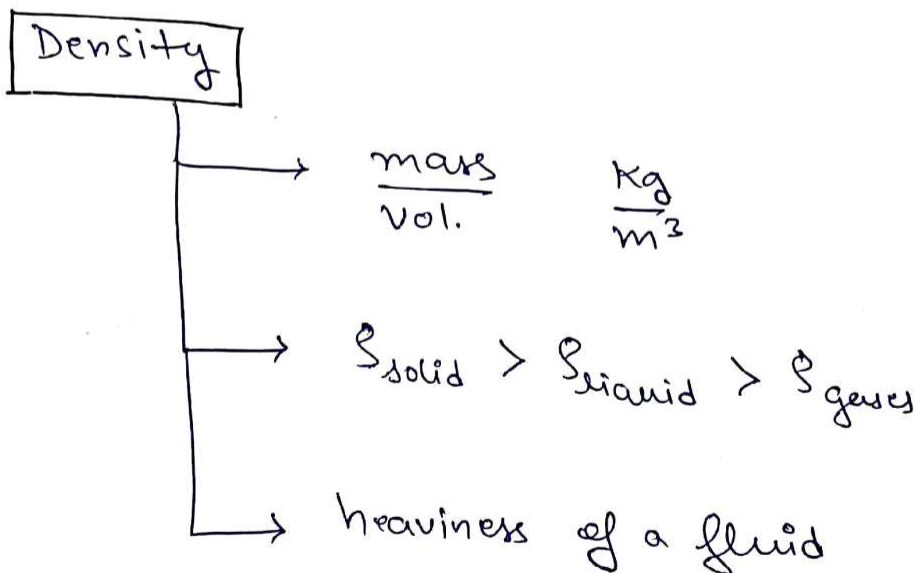
Note: Cohesion and adhesion depends upon the nature of surfaces in contact.

## Fluid Properties

- Properties are certain measurable characteristics that can be quantified.
- With the help of Properties we can identify the fluid.

### 1. Density or mass density ( $\rho$ )

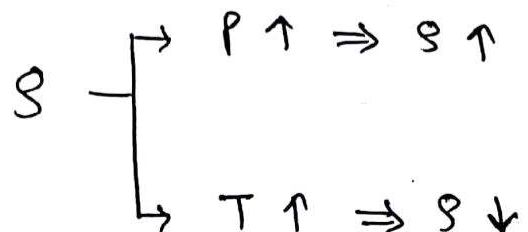
- Density is defined as the ratio of mass per unit volume and its SI units is  $\text{kg/m}^3$ .
- Density basically represents the number of molecules of a fluid in a given volume.
- more the no. of molecules more is the mass & heavier is the fluid and hence density can also be written as representative of heaviness of a fluid.



$$\rho_{\text{H}_2\text{O}} = 10^3 \text{ kg/m}^3$$

$$\rho_{\text{Hg}} = 13.6 \times 10^3 \text{ kg/m}^3$$

$$\rho_{\text{air}} = 1.2 \text{ kg/m}^3$$



## 2. Specific weight or weight density ( $\omega$ )

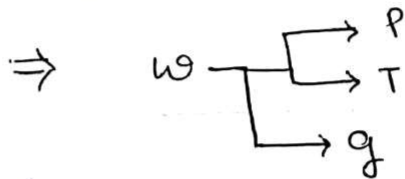
$$\omega = \frac{\text{weight}}{\text{Volume}} = \frac{mg}{V} = \rho g \quad \frac{N}{m^3}$$

$$\omega_{H_2O} = 10^3 \times 9.81 = 9810 \text{ N/m}^3$$

$$\omega_{Hg} = 13.6 \times 10^3 \times 9.81 \text{ N/m}^3$$

weight = specific weight  $\times$  volume

$$\star \boxed{\text{wt.} = \rho g V} \star$$



- Specific weight is defined as weight of fluid per unit volume.
- It basically represents the force exerted by the fluids due to gravity in a given volume.

Note:- density is an absolute quantity w.r.t. location whereas specific wt is a variable quantity w.r.t. location.

## 3. Specific gravity (S)

- Specific gravity is defined as the ratio of the density of the fluid to the density of the standard fluid.
- Standard fluid in case liquid is taken as water whereas standard fluid in case of gases is taken as Air.
- Specific gravity basically shows which fluids are heavier than water & which fluids are lighter than water.

$$S = \frac{\rho_{\text{fluid}}}{\rho_{\text{standard fluid}}}$$

Dimensionless

(i)  $S = 0.750$

$$\rho_{\text{fluid}} = 0.750 \times 10^3 = 750 \text{ kg/m}^3$$

$$S = 13.6$$

$$(ii) \rho_{\text{fluid}} = 13.6 \times 1000 = 13600 \text{ kg/m}^3$$

$$(iii) S = 1$$

$$\rho_{\text{fluid}} = 1000 \text{ kg/m}^3 \rightarrow \text{water}$$

$S > 1 \Rightarrow$  fluid is heavier than water

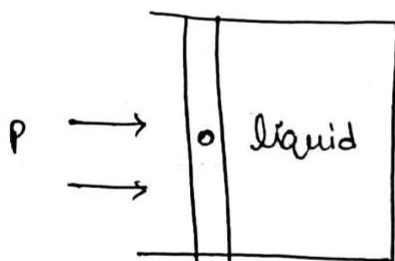
$S < 1 \Rightarrow$  fluid is lighter than water

$S = 1 \Rightarrow$  fluid is water itself.

**Note:** All specific gravities are relative densities but all relative densities are not specific gravities.

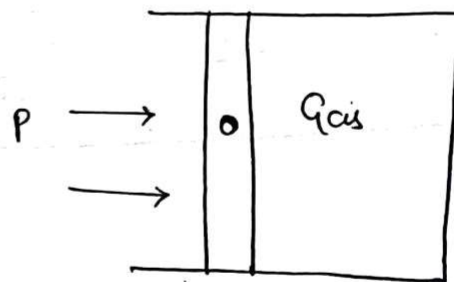
#### 4. Compressibility ( $\beta$ )

If there is a change in  $V$  or  $S$  w.r.t.  $P$  of fluid, it is compressible.



$$P \uparrow, V \times$$

Incompressible



$$P \uparrow, V \downarrow$$

Compressible

$$1 \text{ atm} \rightarrow \rho_{\text{H}_2\text{O}} = 998 \text{ kg/m}^3$$

$$100 \text{ atm} \rightarrow \rho_{\text{H}_2\text{O}} = 1003 \text{ kg/m}^3$$

$$\hookrightarrow \Delta P = 5$$

$$\% \text{ change} = \frac{5}{998} \times 100 \approx 0.5\%$$

- liquids are generally incompressible fluids whereas gases are generally compressible fluids
- with increase in pressure, if variation of volume is large then the fluid is highly compressible
- Mathematically compressibility is defined as reciprocal of bulk modulus of elasticity.

$$\beta = \frac{\Delta V}{V} \Rightarrow m = \beta V \Rightarrow 0 = \beta dV + V d\beta$$

$$\Rightarrow \boxed{-\frac{dV}{V} = \frac{d\beta}{\beta}}$$

$$\beta = \frac{1}{K}$$

$$K = \frac{\text{Hydrostatic stress}}{\text{Volumetric strain}} = \frac{\text{Direct stress}}{\text{Vol. strain}} = \frac{-dP}{dV/V}$$

$$\boxed{K = -V \frac{dP}{dV} = \frac{\beta dP}{d\beta}}$$

$$\boxed{\beta = \frac{1}{K} = -\frac{dV}{V dP} = \frac{d\beta}{\beta dP}} \Rightarrow \text{if } \beta = \text{const w.r.t. } P \Rightarrow \text{fluid is incompressible}$$

- $\beta \propto \frac{1}{K}$  ;  $K_{\text{water}} = 2 \times 10^6 \text{ kPa}$   
 $K_{\text{air}} = 100 \text{ kPa}$

- Note:
- 1) All those fluids are said to be incompressible fluids whose density is constant w.r.t. pressure.
  - 2) Air is generally 20,000 times more compressible than water.
  - 3) if the mach no. of fluid is less than 0.3 it is treated as incompressible.

## Isothermal Bulk modulus

ideal gas

$$PV = nRT$$

$$P = \frac{nRT}{V}$$

$$T = \text{const}$$

$$\frac{dP}{dV} = -\frac{nRT}{V^2}$$

$$K_T = -V \frac{dP}{dV} = \frac{nRT}{V} = P$$

$$\boxed{K_T = P}$$

## Adiabatic bulk modulus

ideal gas, adiabatic, reversible

$$PV^\gamma = \text{const.} = c$$

$$P = c V^{-\gamma} = c \left(\frac{m}{\rho}\right)^{-\gamma} = \frac{c}{m^\gamma} \rho^\gamma$$

$$\frac{dP}{d\rho} = \gamma \frac{c}{m^\gamma} \rho^{\gamma-1}$$

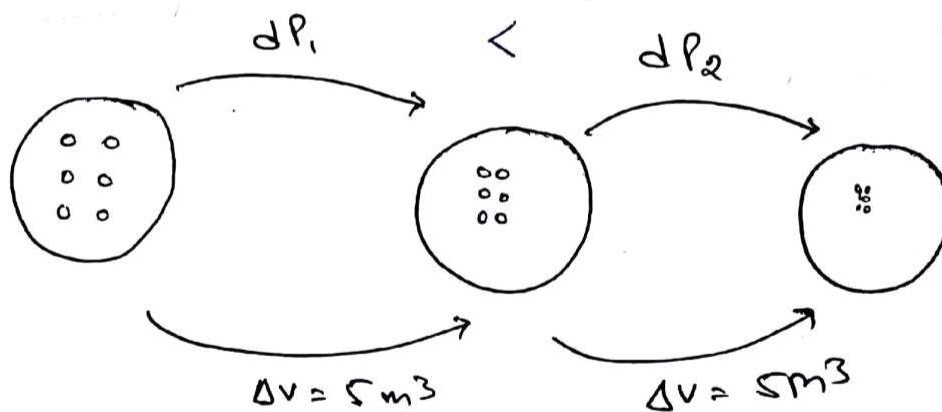
$$K_a = \rho \frac{dP}{d\rho} = \gamma \frac{c}{m^\gamma} \rho^\gamma = \gamma P$$

$$\boxed{K_a = \gamma P}$$

• As  $\gamma$  is always greater than 1 for an ideal gas

$$\boxed{K_a > K_T}$$

$$\Rightarrow \boxed{P \uparrow \Rightarrow K \uparrow \Rightarrow \beta \downarrow}$$



$$K_1 < K_2$$

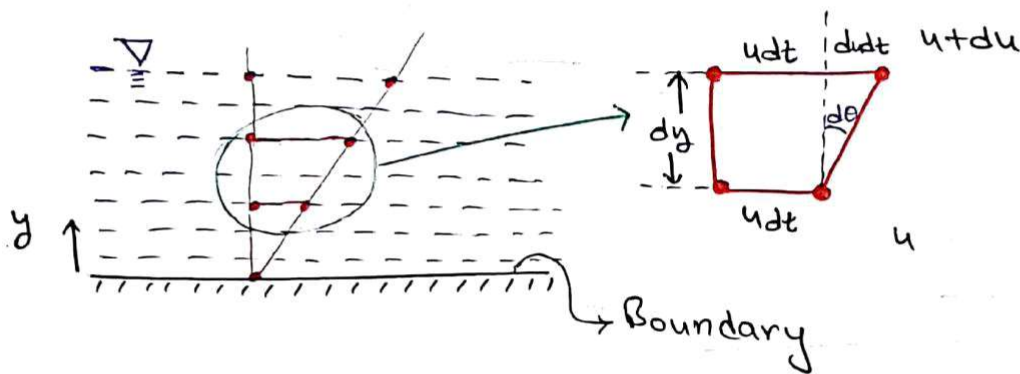
Note: 1) Compressing a gas adiabatically is more difficult than compressing a gas isothermally because while adiabatic compression due to increase in temperature the randomness of molecules increases and hence it provides resistance to compression.



2) with increase in pressure, the Bulk modulus of a gas increases bcz due to increase in pressure molecules will come closure to each other and provide resistance to further compression & hence compressibility decreases.

### 5) Viscosity

viscosity is defined as the internal resistance offered by one layer of the fluid to the adjacent layer.



$$\tan(d\theta) = \frac{du dt}{dy}$$

$$d\theta = \frac{du dt}{dy}$$

$$\boxed{\frac{d\theta}{dt} = \frac{du}{dy}}$$

$$\frac{d\theta}{dt} = \text{Rate of deformation}$$

$$\frac{du}{dy} = \text{Velocity gradient}$$

• Solids  $\rightarrow$  Stress  $\propto$  Strain

fluids  $\rightarrow$  Shear stress  $\propto$  Rate of shear strain  
 Rate of deformation

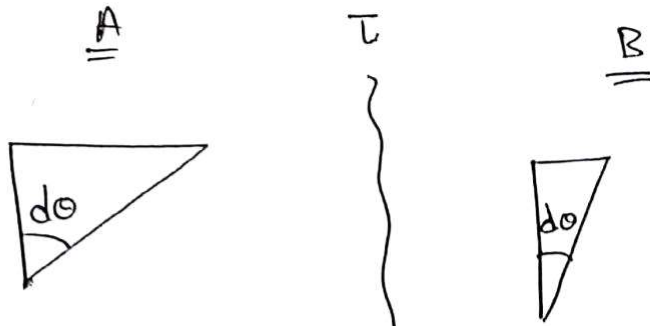
$$\tau \propto \frac{d\theta}{dt}$$

$$\boxed{\tau = \mu \frac{du}{dy}}$$

$\mu =$  viscosity

= Dynamic viscosity

= coefficient of viscosity



•  $\frac{d\theta}{dt}$  is more

•  $\mu$  is less

• Resistance to flow is less

• Flow is easy

•  $\frac{d\theta}{dt}$  is less

•  $\mu$  is more

• Resistance to flow is more

• Flow is difficult

Note: 1) The main reason of viscosity in case of liquid is the molecular bonding whereas in case of gases it is the molecular collision.

2) viscosity basically shows resistance to motion & hence if the viscosity is less the flow is easy.

### Newton's law of viscosity

All those fluids are said to be newtonian fluids for whom shear stress is directly proportional to rate of shear strain.

Shear stress  $\propto$  Rate of shear strain

$$\tau \propto \frac{d\theta}{dt}$$

→ slope = const.

$$\tau = \mu \frac{dy}{dy}$$

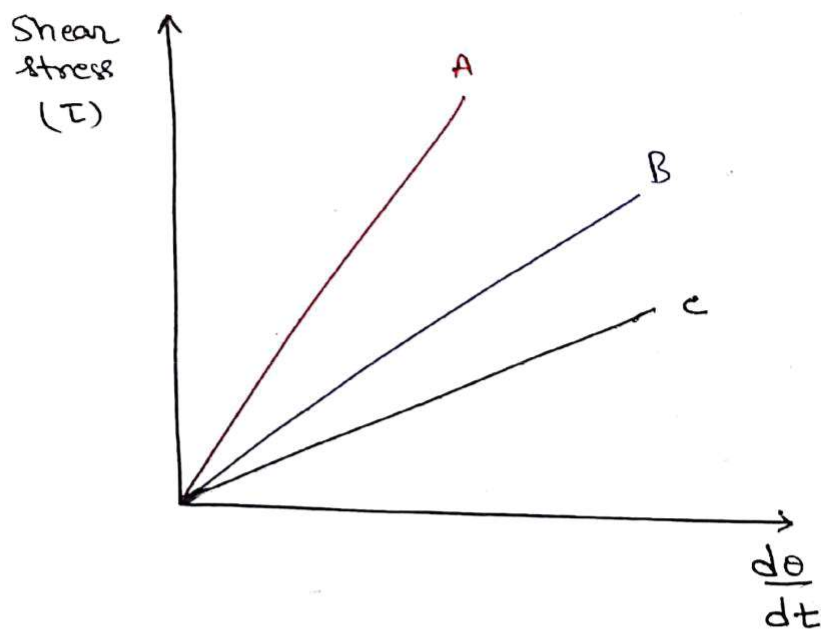
⇒ Newton law of viscosity

⇒ Newtonian fluids

ex.:- water

- Air
- Hg
- Petrol
- Diesel

$\mu = \text{constant w.r.t. } \frac{d\theta}{dt} = \text{slope of } \tau \text{ vs } \frac{d\theta}{dt} \text{ graph}$



A, B, C all are newtonian fluids.

$$\mu_A > \mu_B > \mu_C$$

Note: 1) For a newtonian fluid viscosity is constant w.r.t. deformation.

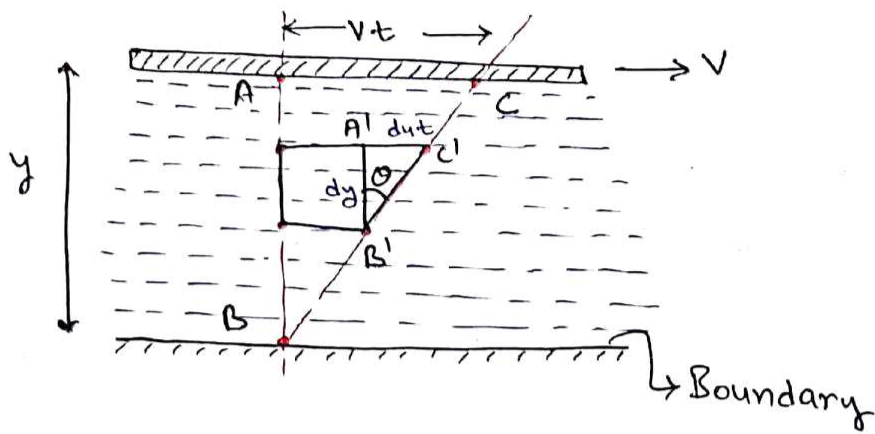
2) For a newtonian fluid, the relation b/w shear stress and rate of deformation is constant linear.

3) liquid  $\rightarrow T \uparrow \Rightarrow \mu \downarrow$   
gases  $\rightarrow T \uparrow \Rightarrow \mu \downarrow$

### Variation of viscosity ( $\mu$ ) w.r.t. temperature:

- In case of liquids with increase in temp<sup>r</sup> the viscosity decreases bcz the main reason of viscosity in case of liquids is molecular bonding and with increase in temp<sup>r</sup> molecular breaks down & viscosity decreases.
- In case of gases the main reason of viscosity is molecular collision & due to this with increase in temp<sup>r</sup> the molecular collision increases and hence viscosity increases.

# Shear force required for linear velocity profile



$$\tan \theta = \frac{vt}{y}$$

$$\tan \theta = \frac{du \cdot t}{dy}$$

⇓

compare

$$\frac{v \cdot t}{y} = \frac{du \cdot t}{dy}$$

$$\boxed{\frac{v}{y} = \frac{du}{dy}}$$

$$\tau = \mu \frac{du}{dy}$$

$$\Rightarrow \tau = \frac{\mu v}{y}$$

$$\boxed{F = \frac{\mu A v}{y}}$$

F = shear force required

$\mu$  = Dynamic viscosity

A = Surface area of plate in contact of fluid

v = Velocity of plate

y = thickness of fluid film.

## Units of viscosity ( $\mu$ )

$$F = \frac{\mu A v}{y} \Rightarrow \mu = \frac{F y}{A v} = \frac{N \cdot m}{m^2 \cdot m \cdot s} = \frac{N \cdot s}{m^2}$$

$$\mu \Rightarrow \text{Pa} \cdot s$$

$$\mu \Rightarrow \frac{kg \cdot m}{s^2} \cdot \frac{s}{m^2} \Rightarrow \frac{kg}{m \cdot s}$$

## Kinematic viscosity ( $\nu$ )

- Kinematic viscosity is defined as the ratio of dynamic viscosity to density.

$$\boxed{\nu = \frac{\mu}{\rho}} \quad \frac{\text{kg}}{\text{m-s}} \times \frac{\text{m}^3}{\text{kg}} \Rightarrow \frac{\text{m}^2}{\text{s}}$$

- Dynamic viscosity shows resistance to motion whereas kinematic viscosity shows resistance to molecular momentum transfer [molecular collision]  
[only valid for liquids]

### Note

$$\text{Dynamic viscosity } (\mu) \Rightarrow \frac{\text{kg}}{\text{m-s}} \Rightarrow \frac{1000 \text{ gm}}{100 \text{ cm-sec}} \Rightarrow 10 \frac{\text{gm}}{\text{cm-sec}} \rightarrow \text{Poise}$$

$$\Rightarrow \boxed{1 \frac{\text{kg}}{\text{m-s}} = 10 \text{ Poise}}$$

$$\text{Kinematic viscosity } (\nu) \Rightarrow \frac{\text{m}^2}{\text{s}} \Rightarrow 10000 \frac{\text{cm}^2}{\text{sec}} \Rightarrow \boxed{1 \frac{\text{m}^2}{\text{s}} = 10^4 \text{ Stokes}}$$

as temp  $\uparrow$

liquids  $\Rightarrow \mu \downarrow$

$$\nu = \frac{\mu \downarrow}{\rho = \text{const}} \Rightarrow \nu \downarrow$$

Gas  $\Rightarrow \mu \uparrow$

$$\nu = \frac{\mu \uparrow}{\rho \downarrow} \Rightarrow \nu \uparrow \text{ faster rate}$$

T ↑

	$\mu$	$\nu$
liquid	↓	↓
Gas	↑	↑ faster rate

# for ideal gas

$$PV = nRT$$

$$P = \rho RT \Rightarrow \rho \propto \frac{1}{T}$$

• Sutherland's eqn

$$\mu_{\text{gas}} \propto \sqrt{T}$$

$$\nu_{\text{gas}} = \frac{\mu_{\text{gas}}}{\rho_{\text{gas}}} \Rightarrow \frac{\sqrt{T}}{1/T} \Rightarrow T^{3/2}$$

$$\Rightarrow \nu_{\text{gas}} \propto T^{3/2}$$

## Non-Newtonian fluids [ESE objective only]

- Non-newtonian fluids are the one whose viscosity is going to vary with rate of deformation
- In non-newtonian fluids the variation of shear stress with  $\frac{d\sigma}{dt}$  is non-linear
- The study of non-newtonian fluids is known as Rheology

$$\tau = A \left( \frac{d\gamma}{dt} \right)^n + B$$

General equation

• For newtonian fluid

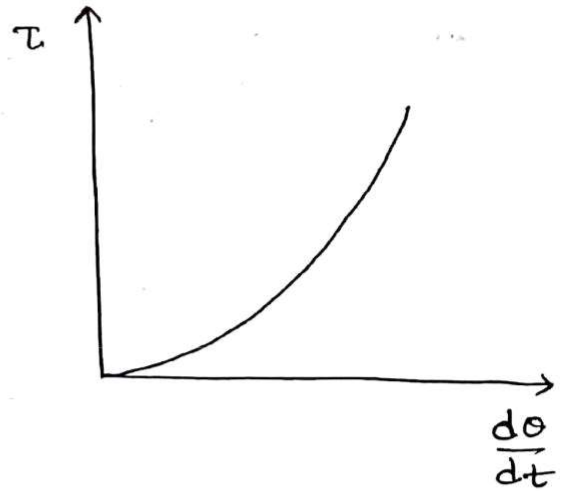
$$B = 0$$

$$n = 1$$

A)  $\mu$  varies with  $\frac{d\theta}{dt}$

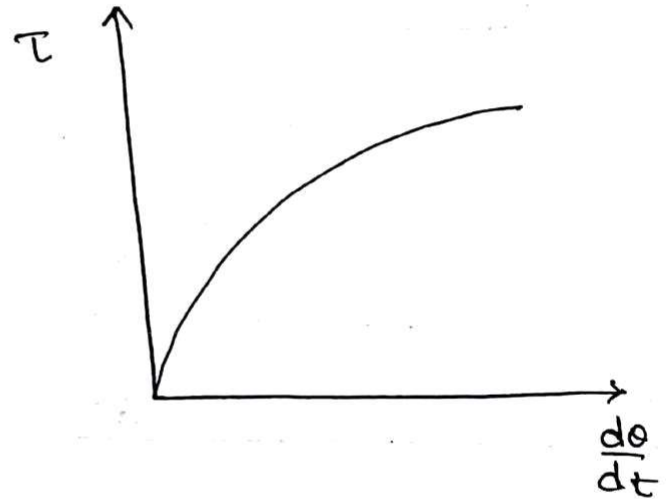
1. Dilatant fluid ( $n > 1, B = 0$ )

- $\mu \uparrow$  with  $\frac{d\theta}{dt}$
- Shear thickening fluids
- ex:- Sugar in water solution  
- Rice starch



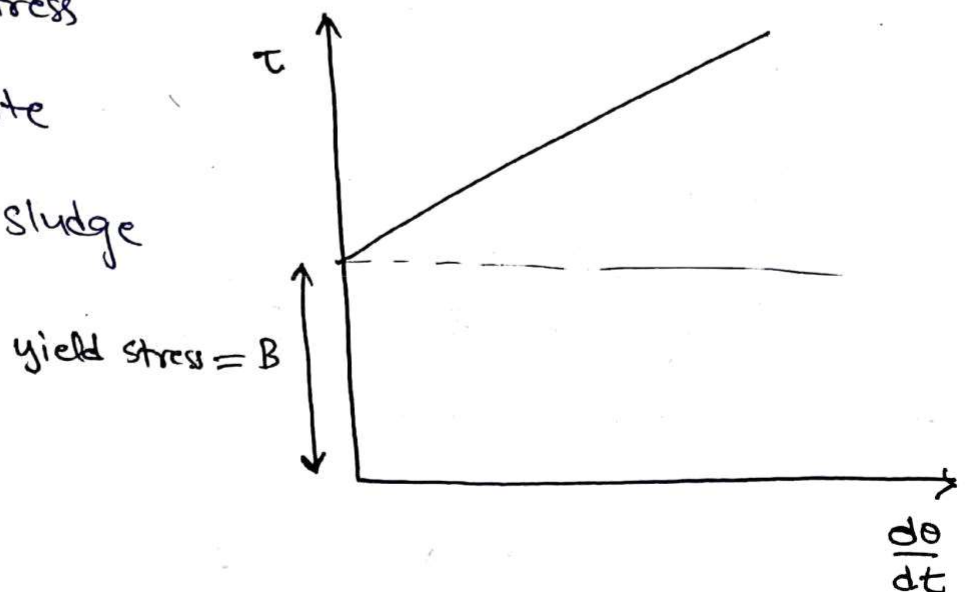
2. Pseudoplastic ( $n < 1, B = 0$ )

- $\mu \downarrow$  with  $\frac{d\theta}{dt}$
- Shear thinning fluids
- ex:- Paint  
- blood  
- lipstick



3. Bingham Plastic ( $n = 1, B \neq 0$ )

- $B =$  yield stress
- ex:- tooth paste  
- Gel  
- sewage sludge



B)  $\mu$  vary with time

1. Rheopectic  $\mu \uparrow$  with time

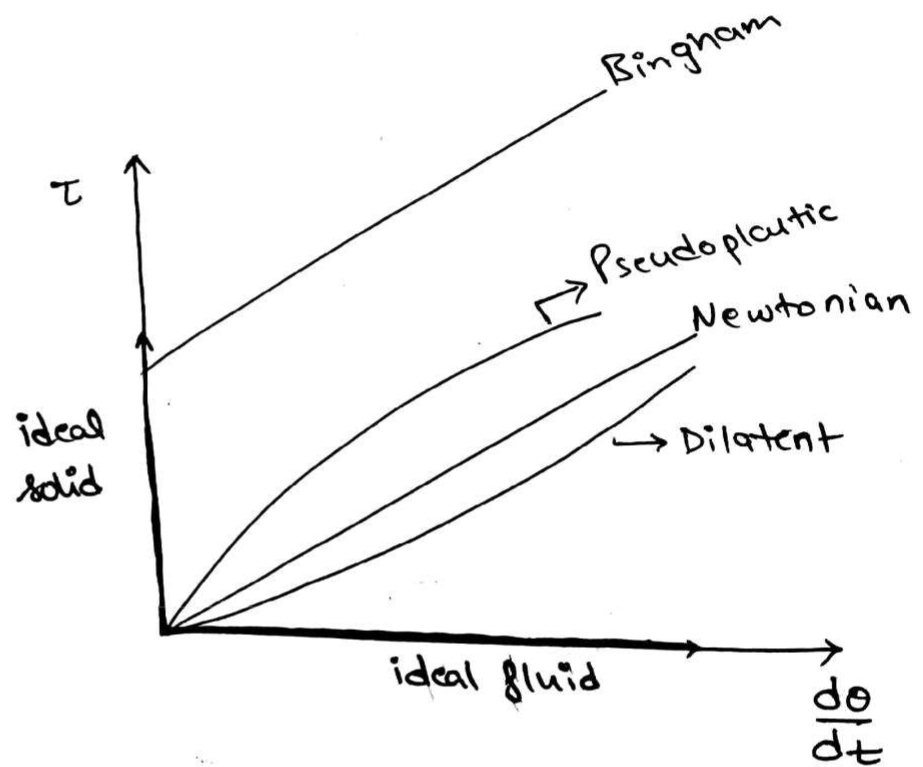
ex:- Gypsum in  $H_2O$   
- Printer ink

2. Thixotropic  $\mu \downarrow$  with time

ex:- ice-cream

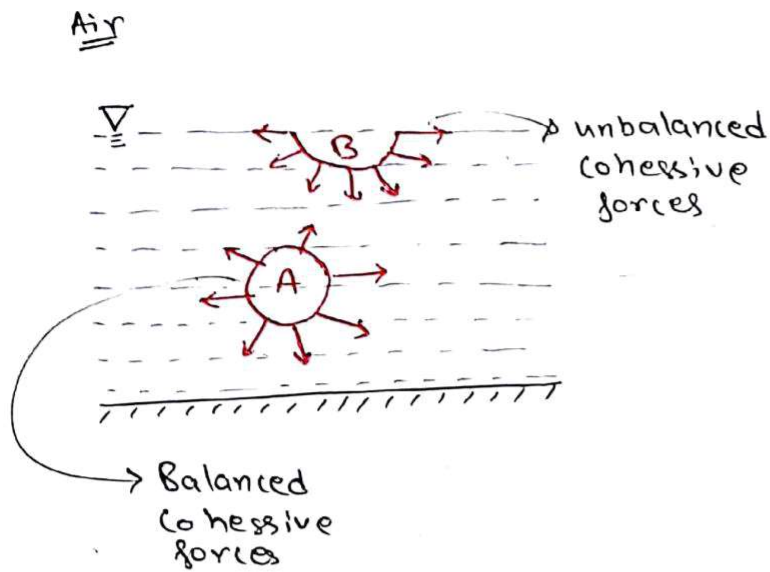
### Ideal fluid

- Non-Viscous
- Incompressible





## 6. Surface tension ( $\sigma$ )



- Let us consider a molecule ~~over~~ <sup>under</sup> the surface of the fluid. This molecule "A" is under the action of balanced cohesive forces from all sides and thus this molecule is under equilibrium.
- Let us consider a molecule "B" which is on the surface of the fluid & this molecule is under the action of net downward pull and hence this molecule is under stretched condition.
- There are large number of molecules on the surface and all the molecules are under a pull and due to this there appears to be a membrane over the surface of the fluid which can bear small loads. And this property is known as surface tension.
- Surface tension is also defined as a line force with forces acting  $\perp$  to this line in the plane of the surface.
- Surface tension is also given as force acting per unit length and its SI unit is  $N/m$ .

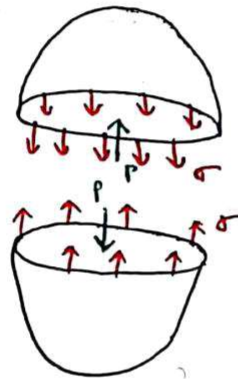
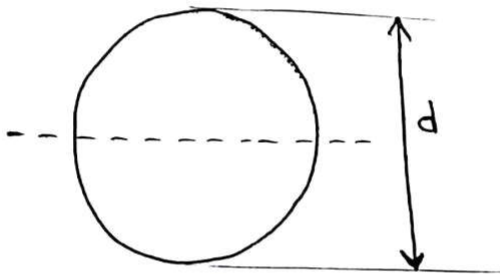
$$\sigma = \frac{F}{L}$$

$N/m$

$F$  = Surface tension force  
 $L$  = length over which surface tension is acting [Perimeter]

- with increase in temp<sup>r</sup> surface tension decreases. due to breaking of cohesive bonds
- At critical point as there is no interface, surface tension is zero.
- Detergents are used while washing cloths to reduce surface tension and break the fluid film so that dirt particles can come out easily
- liquid droplet takes up the shape of a sphere due to surface tension.

⇒ Excess Pressure inside a liquid droplet

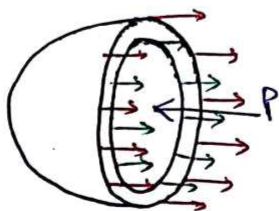


$$F_{\text{pressure}} = F_{\text{surface tension}}$$

$$P \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$\Rightarrow \boxed{P_{\text{gauge}} = \frac{4\sigma}{d}}$$

⇒ Excess Pressure inside a bubble



$$F_{\text{pressure}} = F_{\text{surface tension}}$$

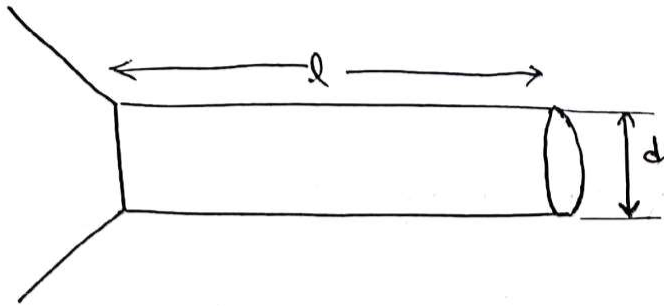
$$P \times \frac{\pi}{4} d_i^2 = \sigma \times (\pi d_i + \pi d_o)$$

$$d_i \approx d_o \approx d$$

$$P \times \frac{\pi}{4} d^2 = \sigma \times 2\pi d$$

$$\Rightarrow \boxed{P_{\text{gauge}} = \frac{8\sigma}{d}}$$

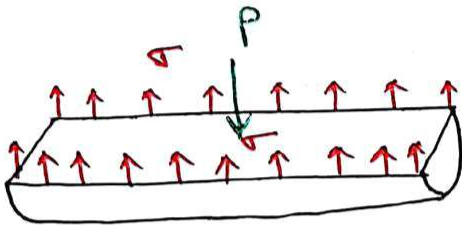
⇒ Excess pressure inside liquid jet



$$F_{\text{pressure}} = F_{\text{surface tension}}$$

$$P \times D \times l = \sigma \times 2l$$

$$\Rightarrow \boxed{P_{\text{gauge}} = \frac{2\sigma}{d}}$$

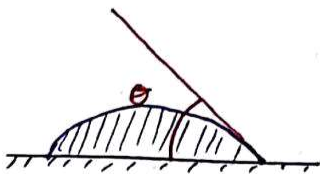


## 7. Capillarity

- Capillarity is defined as the rise or fall of liquid when a small diameter glass tube is inserted in it is known as known capillarity.
- The main reason of capillary rise is Adhesion.  
ex:-  $H_2O$  in glass tube
- The main reason of capillary fall is cohesion  
ex:-  $Hg$  in glass tube.

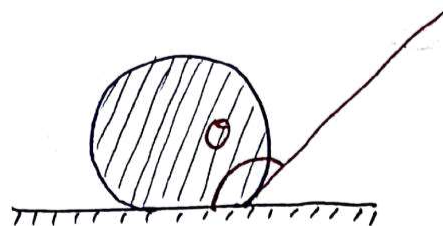
## Angle of contact ( $\theta$ )

### wetting fluids



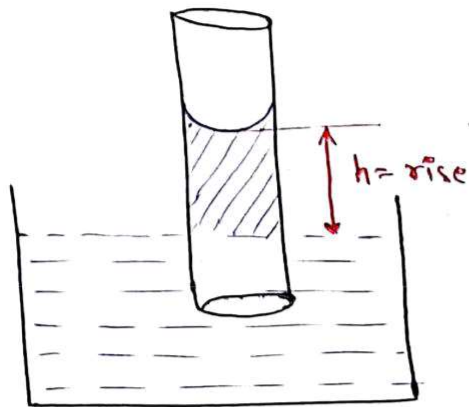
- Adhesion is more
- $\theta < 90^\circ$
- ex:- glass & water

### non-wetting fluids



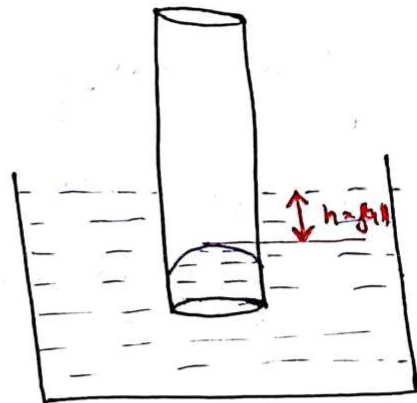
- Cohesion is more
- $\theta > 90^\circ$
- eg:- glass & mercury

## Capillary rise



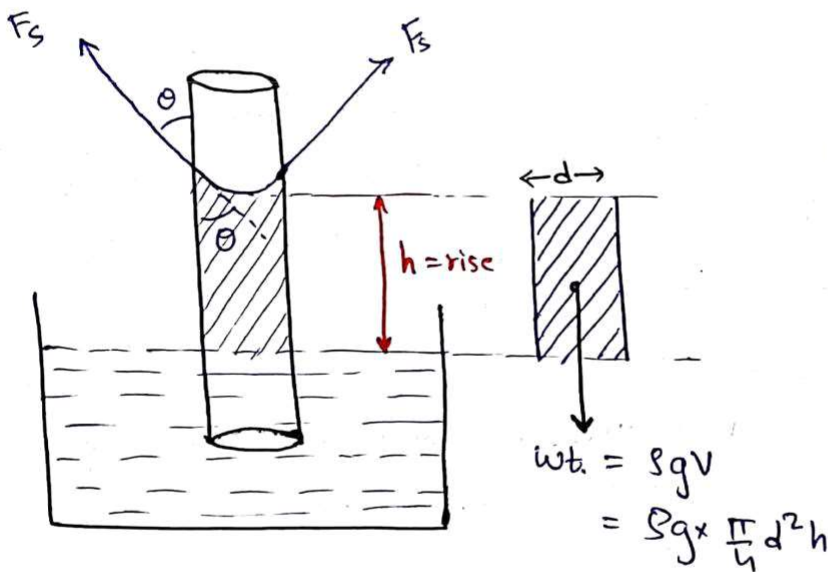
- Adhesion
- $H_2O$  & glass tube

## capillary fall



- cohesion
- Hg & glass tube

⇒ Expression for capillary rise in tube



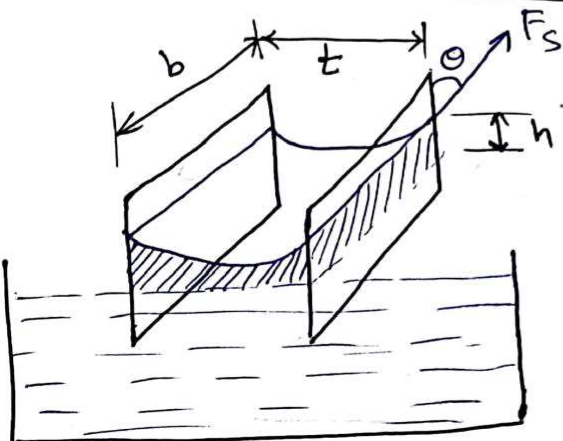
$$F_s \cos \theta = \text{wt. of fluid}$$

$$\sigma \times d \cos \theta = \rho g \times \frac{\pi}{4} \times d^2 h$$

$$\sigma \times \pi d \cos \theta = \rho g \frac{\pi}{4} d^2 h$$

$$\Rightarrow \boxed{h = \frac{4\sigma \cos \theta}{\rho g d}}$$

⇒ Expression for capillary rise b/w flat parallel plates



$$F_s \cos \theta = \text{wt. of fluid}$$

$$\sigma \times 2b \cos \theta = \rho g \times b t h$$

$$\Rightarrow \boxed{h = \frac{2\sigma \cos \theta}{\rho g t}}$$

Note: 1)  $h \propto \frac{1}{d}$

So for same conditions if diameter is changed

$$h_1 d_1 = h_2 d_2$$

2) To neglect the capillary rise or fall

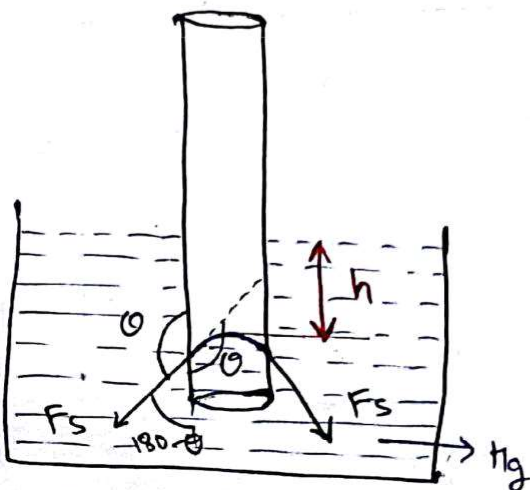
$$d > 10 \text{ mm}$$

From the above we can conclude that capillary rise is inversely proportional to diameter of the tube and to neglect the effect of capillary, ex: in case of pressure measurement the diameter of glass tube must be taken greater than 10 mm

4) liquid droplets takes up the shape of sphere bcz surface tension will try to minimise the surface area whereas pressure will resist this. Due to decrease in surface area the pressure inside the droplet increase and it keeps on increasing until both come in equilibrium.

5) Pressure & surface tension force will come in equilibrium in such a manner that the droplet assume spherical shape which is having minimum surface area according to mathematics.

⇒ Expression for capillary fall in glass tube

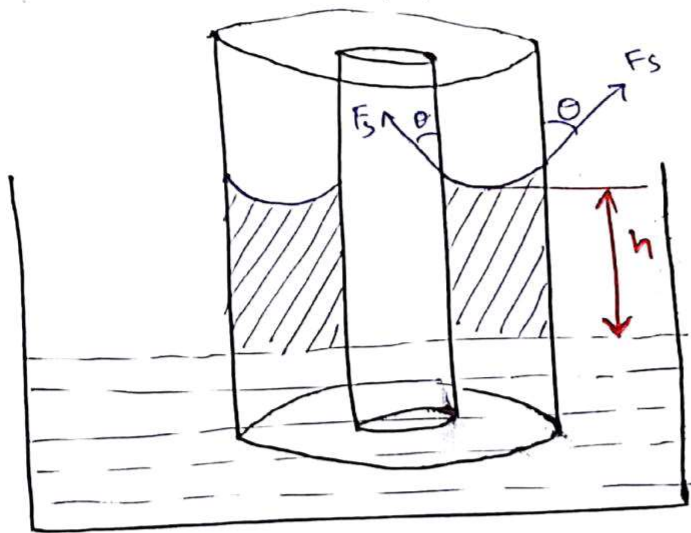
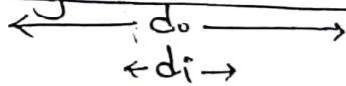


$$F_{pr} = F_{\text{surface tension}} \times \cos(180 - \theta)$$

$$Sg h \times \frac{\pi}{4} d^2 = -\sigma \times \pi d \times \cos \theta$$

$$h = \frac{-4\sigma \cos \theta}{Sg d}$$

⇒ Capillary rise in an annular space



$$\text{wt. of fluid} = \rho g V$$

$$= \rho g \times \frac{\pi}{4} (d_o^2 - d_i^2) h$$

$$F_c = \sigma \times (\pi d_o + \pi d_i) \times \cos \theta$$

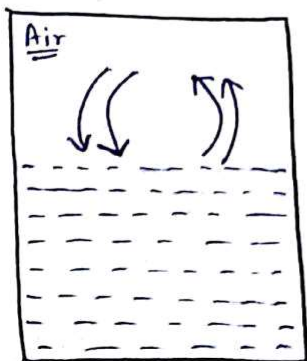
$$\cancel{\pi} \times \cancel{\sigma} (\cancel{d_o} + \cancel{d_i}) \cos \theta = \rho g \times \cancel{\frac{\pi}{4}} (\cancel{d_o} + \cancel{d_i}) (d_o - d_i) h$$

$$\Rightarrow h = \frac{4 \sigma \cos \theta}{\rho g (d_o - d_i)}$$

Note :-

value of $\theta$	combination
$32^\circ$	glass & water
$132^\circ$	glass & Hg
$0^\circ$	pure water & glass

## 8. Vapour Pressure



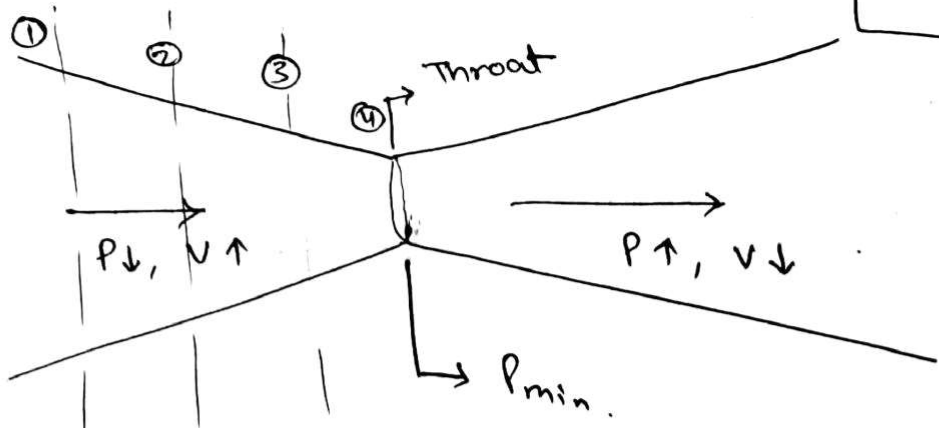
- Let us consider a closed container that is partially filled with fluid.
- The molecules on the surface of the fluid are in highly excited state and by taking energy from the molecules beneath it these molecules evaporate.

- The air above the fluid has some absorption capacity and this absorption capacity is known as saturation.
- Under saturation conditions the number of molecules leaving the surface of the fluid on evaporation becomes equal to number of molecules rejoining the surface on condensation.
- The force exerted by the vapours under saturation condition over the surface of the fluid is known as saturated vapour pressure or vapour pressure ( $P_v$ ).

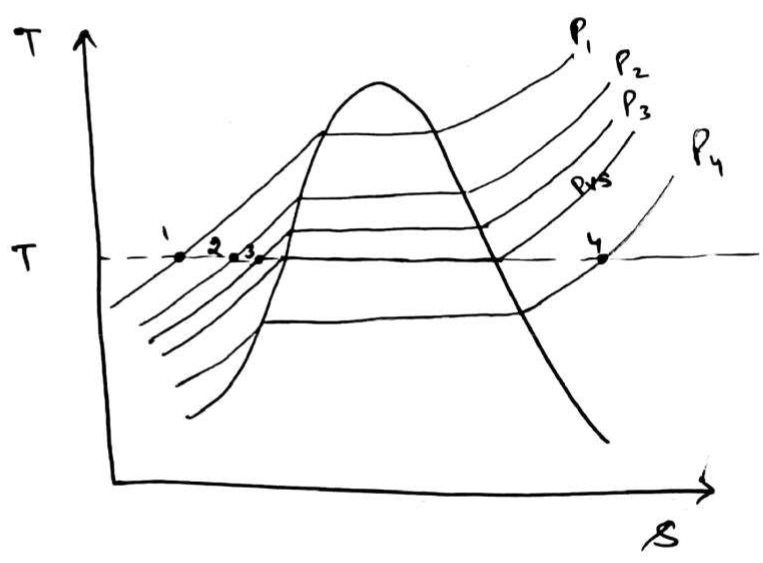
### Cavitation

- If the pressure of the flowing fluid at any point in the fluid flow becomes less than the saturated vapour pressure at that given temp<sup>r</sup> boiling will start and bubbles are formed. When these bubbles move to the high pressure region, the high pressure fluids collapses these bubbles and strikes with the wall and this leads to high order wear & tear and noise and this phenomenon is known as cavitation.
- Cavitation is a low pressure phenomenon.
- With increase in temp<sup>r</sup> vapour pressure increases.
- Highly volatile fluids have higher vapour pressure at a given temp<sup>r</sup>.
- Hg is having lowest vapour pressure because its cohesive bonds are strong and evaporation is less.
- Due to the above reason Hg is used as a manometric fluid in pressure measurements.

Monday  $\Rightarrow$  8 to 4 PM



- If  $P_{min} > P_{vs} \Rightarrow$  flow is normal
- If  $P_{min} < P_{vs} \Rightarrow$  Boiling  $\Rightarrow$  Bubbles  $\Rightarrow$  Cavitation





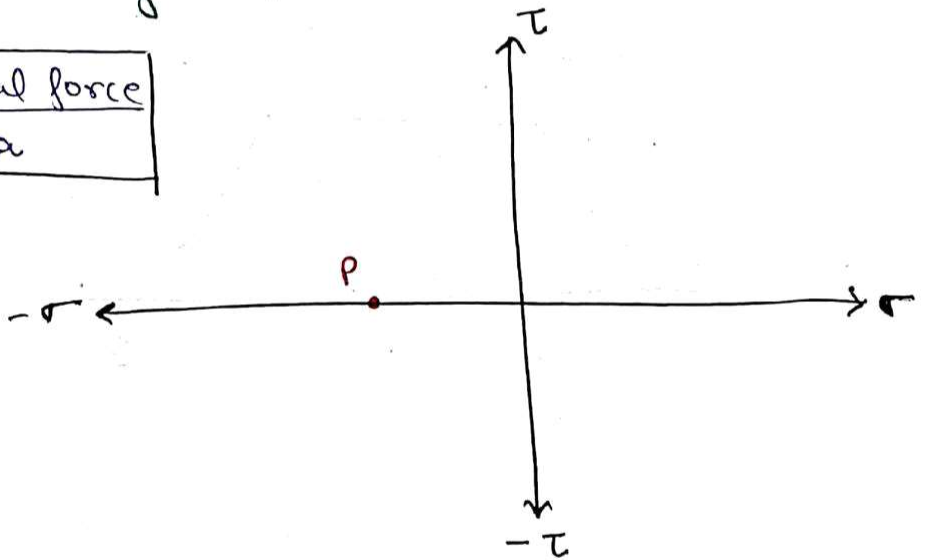
## CHAPTER - 2

### FLUID STATICS

#### A) PRESSURE MEASUREMENT

- Pressure is defined as normal force per unit area and its SI unit is  $N/m^2$  or Pa.
- Pressure basically represents the number of molecules striking a surface, if there are no molecules then there will be no pressure and hence pressure is a representative of molecules.
- Pressure is always compressive in nature and it is shown by a point on Mohr's diagram.

$$\text{Pressure} = \frac{\text{Normal force}}{\text{Area}}$$



Note: Due to striking of molecules there is a change in momentum and due to that force is created.

#### Types of Pressure

##### 1. Atmospheric Pressure ( $P_{atm}$ )

Atmospheric pressure is the pressure exerted by environmental air. Atmospheric pressure is measured by a device known as barometer. It was invented by Toricelli.

## 2. Gauge Pressure ( $P_{\text{gauge}}$ )

- Gauge Pressure are the pressures that are measured w.r.t. atmospheric Pressure.
- While measuring gauge pressure atmospheric pressure is taken as zero.
- Gauge Pressure can be +ve or -ve.
- Vacuum pressures or Pressures less than atmospheric are known as -ve gauge pressures

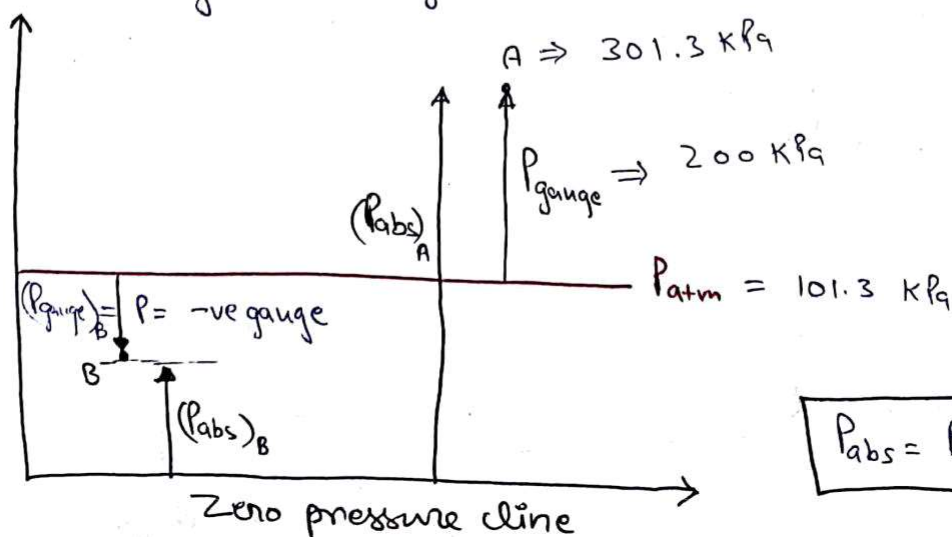
## 3. Absolute Pressure ( $P_{\text{abs}}$ )

- These are the pressures that are measured w.r.t. zero pressure line.
- Absolute pressure are always positive.

## 4) Vacuum Pressures ( $P_{\text{vacuum}}$ )

- All those pressures that are less than atmospheric pressures are known as vacuum pressures.
- Vacuum Pressures are also known as negative gauge pressures.

**Note:** For all numerical Problems we have to find gauge Pressure until and unless absolute pressure is asked.  
• All the negative gauged Pressures are taken and considered with a negative sign.



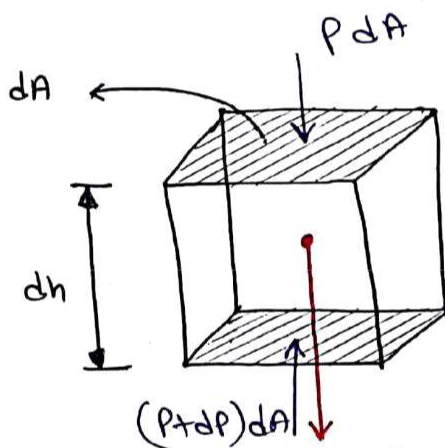
$$P_{\text{abs}} = P_{\text{atm}} + P_{\text{gauge}}$$

$$\begin{aligned}
 1 \text{ atm} &= 1.013 \text{ bar} \\
 &= 101.3 \text{ kPa} \\
 &= 760 \text{ mm Hg} \\
 &= 10.3 \text{ m of water column}
 \end{aligned}$$

## Hydrostatic Law

The variation of pressure in a fluid in vertical dir<sup>n</sup> is directly proportional to specific weight

Air      ↓   ↓   ↓   ↓    $P_{atm}$



$$\text{wt. of fluid} = \rho \cdot g \cdot dA \cdot h$$

$$\sum F = \cancel{m \cdot a}$$

$$\sum F = 0$$

$$\cancel{P dA} + \rho g dA h = \cancel{P dA} + dP dA$$

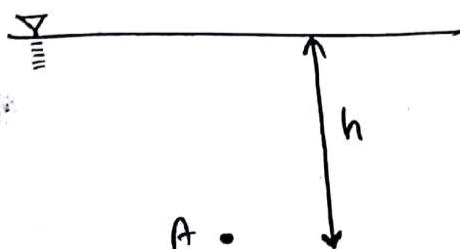
$$\rho g dA h = dP dA$$

$$\frac{dP}{dh} = \rho g$$

$$\boxed{\frac{dP}{dh} = w}$$

## Pressure at a depth "h"

↓↓↓  $P_{atm}$



$$\int dP = \int \rho g dh$$

$$P = \rho g h + C$$

$$\text{at } h=0; P = P_{atm} \Rightarrow C = P_{atm}$$

$$\boxed{P = \rho g h + P_{atm}}$$

$$\boxed{P_{gauge} = \rho g h} \quad \text{N/m}^2$$

- Note:-
- As we move vertically down in a fluid the pressure increase as  $+ \rho g h$ .
  - As we move vertically up in a fluid the pressure decreases as  $- \rho g h$ .
  - There is no change of Pressure in horizontally same level.

### Conversion of one fluid column to another fluid column.

$$\rho_1 g h_1 = \rho_2 g h_2$$

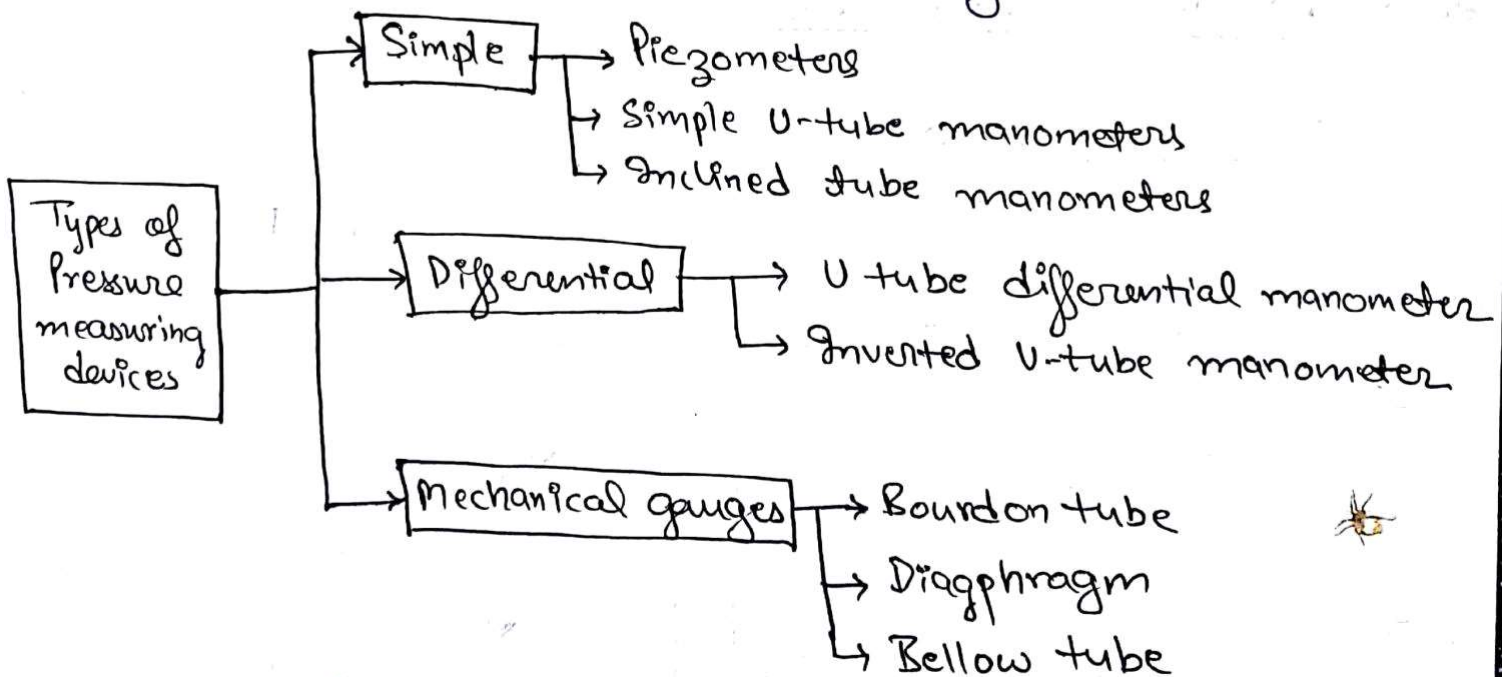
↳ valid for all fluids

$$\rho_1 h_1 = \rho_2 h_2$$

↳ valid for fluids of same nature  
(i.e. liquid - liquid)  
or  
gas - gas

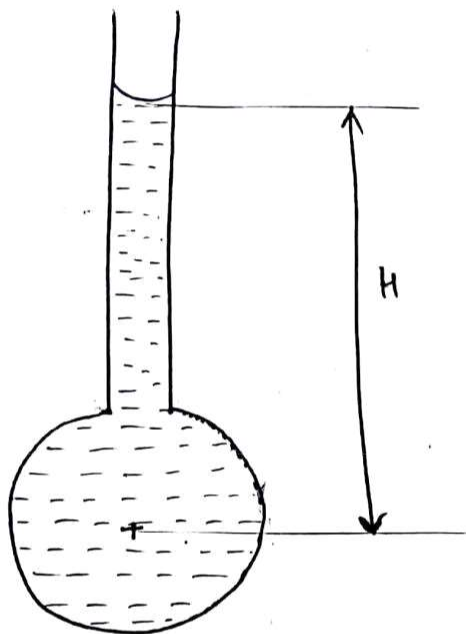
### Pressure measurement

Pressure is measured by Principle of manometry and the devices used for measuring pressure are known as manometers and the branch of science dealing with pressure measurement is known as manometry.



# 1. Piezometers

- A Piezometer is a simple glass tube that is open at both the ends.
- One end of the Piezometer is connected to the pipeline whose pressure needs to be measured and the other end is kept open to the atmosphere.
- Piezometers can't be used to measure very high pressure and gas pressures.



$$P = \rho g H$$

# 2. Simple U-tube manometer

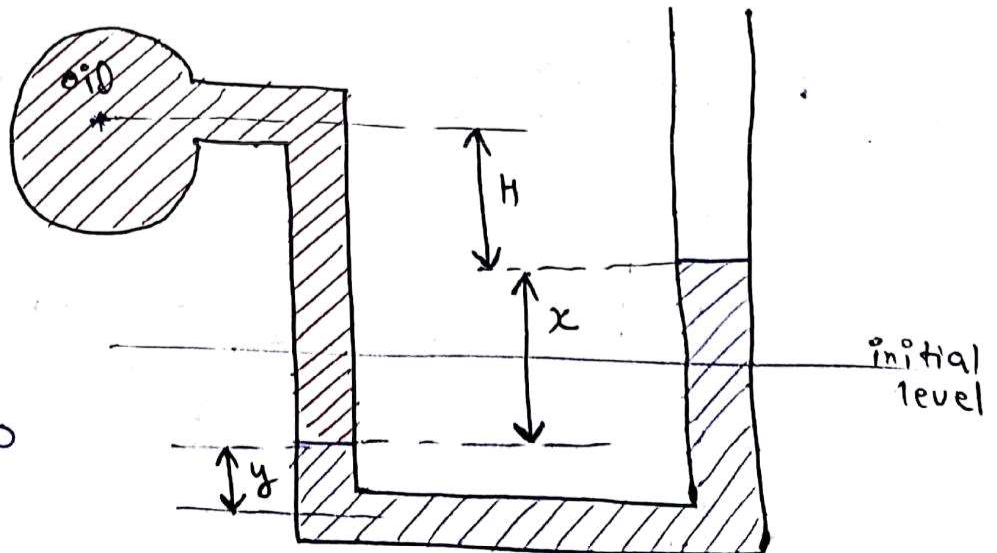
## Jumping Jack method

$$P_0 + \rho_0 g (H+x)$$

$$+ \rho_m g y$$

$$- \rho_m g y$$

$$- \rho_m g x = 0$$



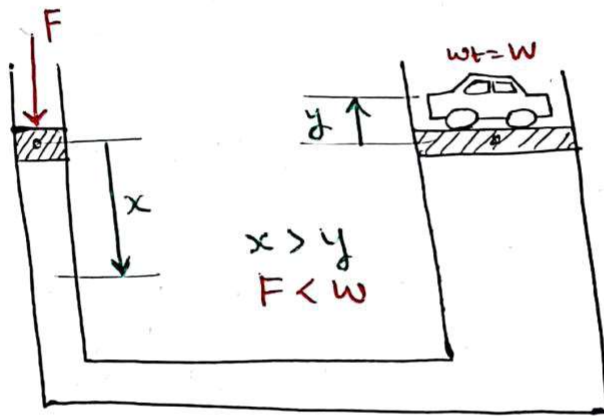
P-9  
01

$$\rho \times 3h + 2\rho \times 1.5h + 3\rho h = 3\rho H$$
$$9h = 3H \Rightarrow \frac{H}{h} = 3$$

## Pascal's Law

According to Pascal's law in a static fluid, the pressure at a point is equally distributed in all the dir<sup>n</sup>. This is known as Pascal's law.

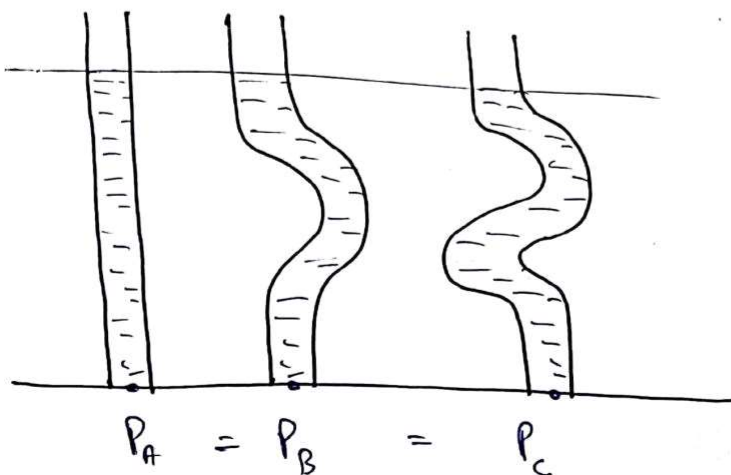
- ex:- hydraulic lift  
- hydraulic break



In a hydraulic lift a smaller force is required to lift a larger weight but still the conservation of energy is not violated bcz the smaller force moves by a larger distance whereas larger wt. moves by smaller distance.

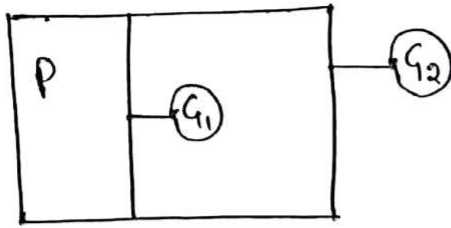
And hence ~~of conservation of~~ work done in both the cases are same and hence conservation of energy is followed.

## Hydrostatic Paradox



Pressure only depends on vertical height of fluid and not on the shape of container or tube.

P-10  
Q4



$$P_{G1} = 5 \text{ bar}$$

$$P_{G2} = 1 \text{ bar}$$

P-10  
Q5

$$P = 225 + 100 + 1.05 \times 9.81 \times 200$$

$$= 2425 \text{ kPa}$$

P-10  
Q6

$$\frac{m}{V} g = 12000$$

$$\rho g = 12000$$

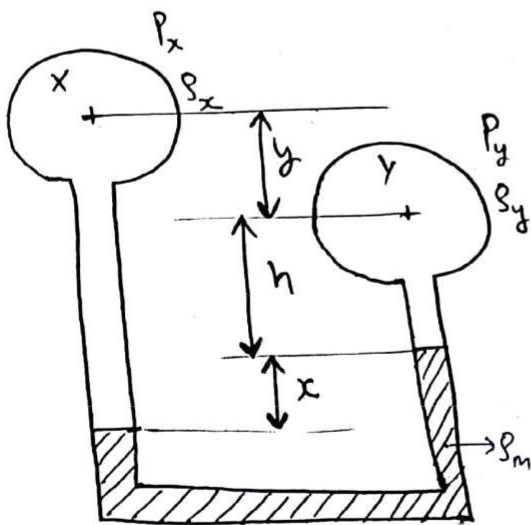
$$\rho = \frac{12000}{g}$$

$$P = \frac{10 \times 10^3 \times 9.81}{g} + \frac{12000}{g} \times 2$$

$$= 98.1 + 24$$

$$= 122.1 \text{ kPa}$$

### 3. Differential U-tube manometer

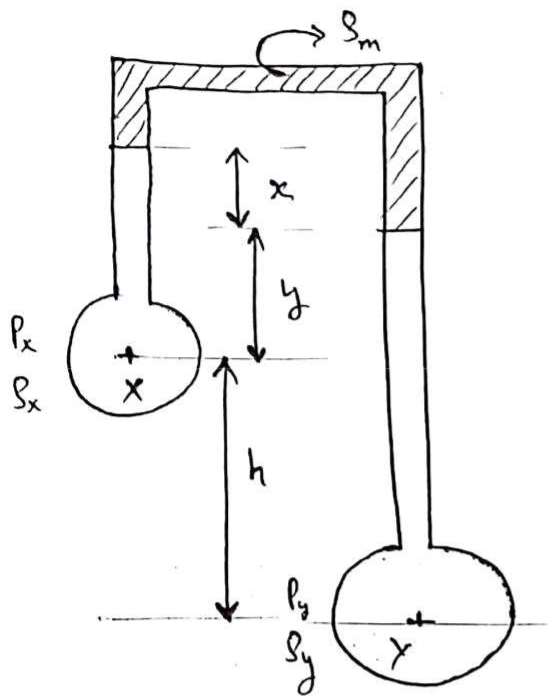


$$P_x + \rho_x g(y+h+x)$$

$$- \rho_m g x$$

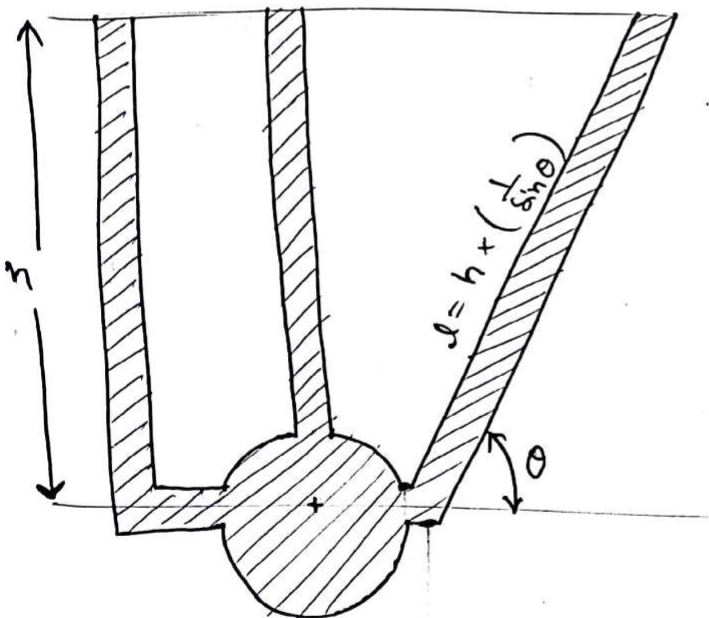
$$- \rho_y g h = P_y$$

#### 4. Differential inverted U-tube manometer



$$P_x - S_x g(y+x) + S_m g x + S_y g(h+y) = P_y$$

#### 5. Inclined tube manometer



$\frac{1}{\sin \theta}$  = Sensitivity of inclined manometer.

$\theta \uparrow \Rightarrow$  Sensitivity  $\downarrow$

- Notes:
- inverted U-tube manometers are used when the pipelines are underground & in these manometer the density of manometric fluid is less than the density of flowing fluid ( $S_m < S$ )
  - multi U-tube manometers are used for measuring high pressures.

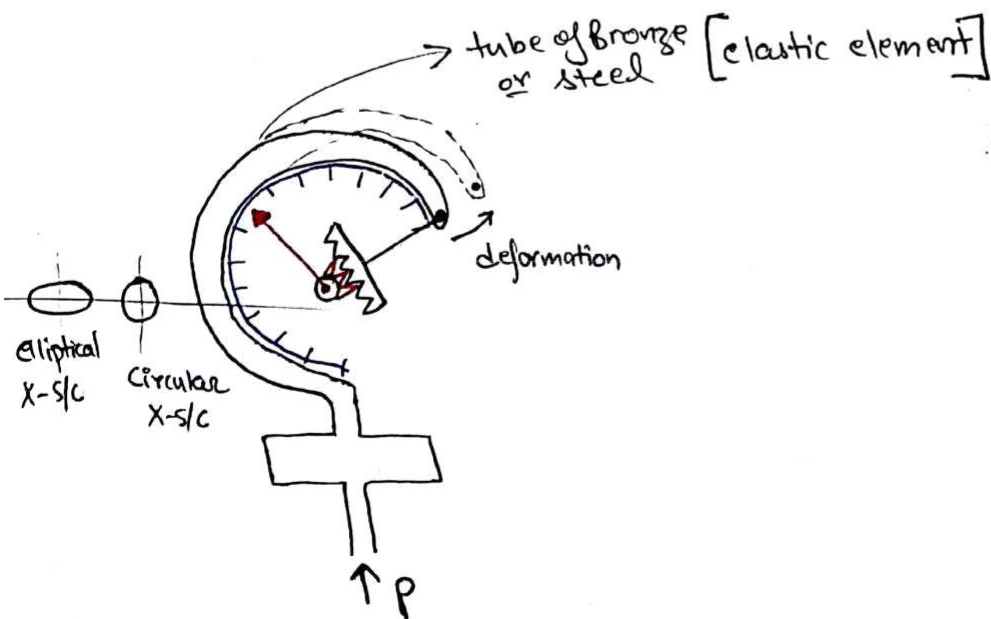


- The limb of the manometer is inclined to increase the sensitivity of a manometer.
- Sensitivity is defined as the smallest pressure diff. that a manometer can measure.
- By inclining the tube the sensitivity increases by a factor of  $\frac{1}{\sin \theta}$
- $\theta \downarrow \Rightarrow$  Sensitivity  $\uparrow$
- inclined tube manometers are used for measuring very small pressure differences in which deflection is very small.

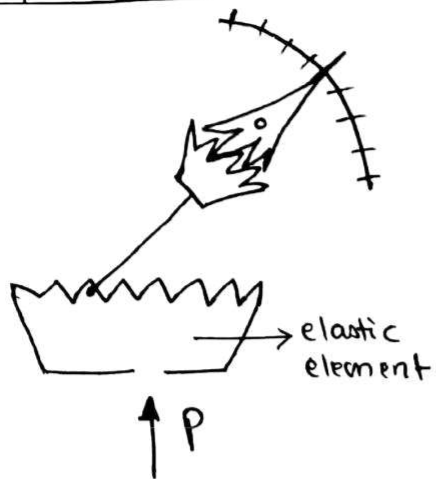
## 6. Mechanical Gauges

- mechanical gauges are used to measure pressure that are very high with less accuracy
- All the mechanical gauges consist of an elastic element which gets deformed on application of pressure.
- This deformation is further magnified by gear & pinion arrangement to a pointer scale & pressure is measured

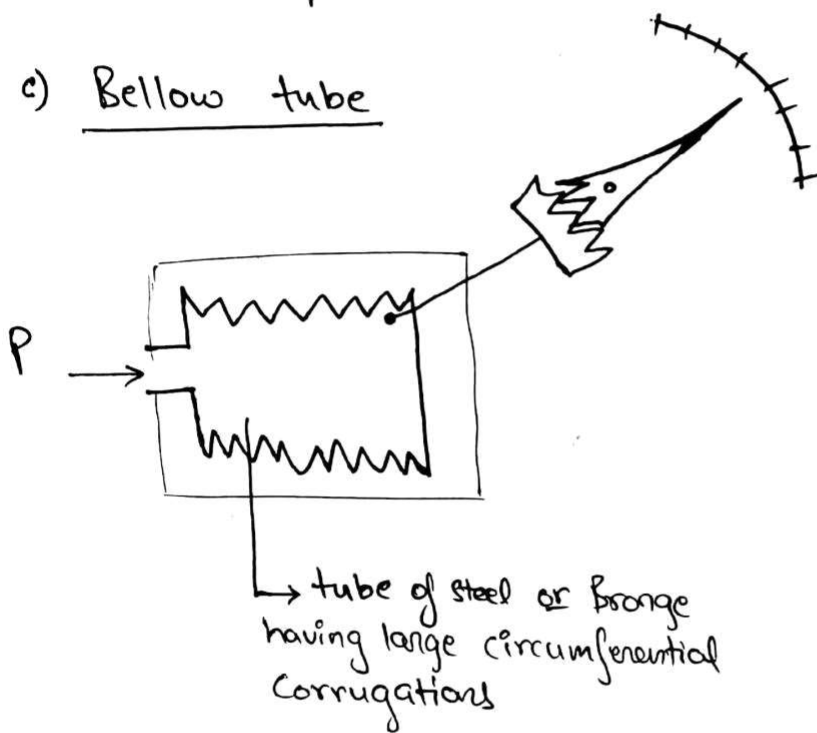
### a) Bourdon tube



b) Diaphragm



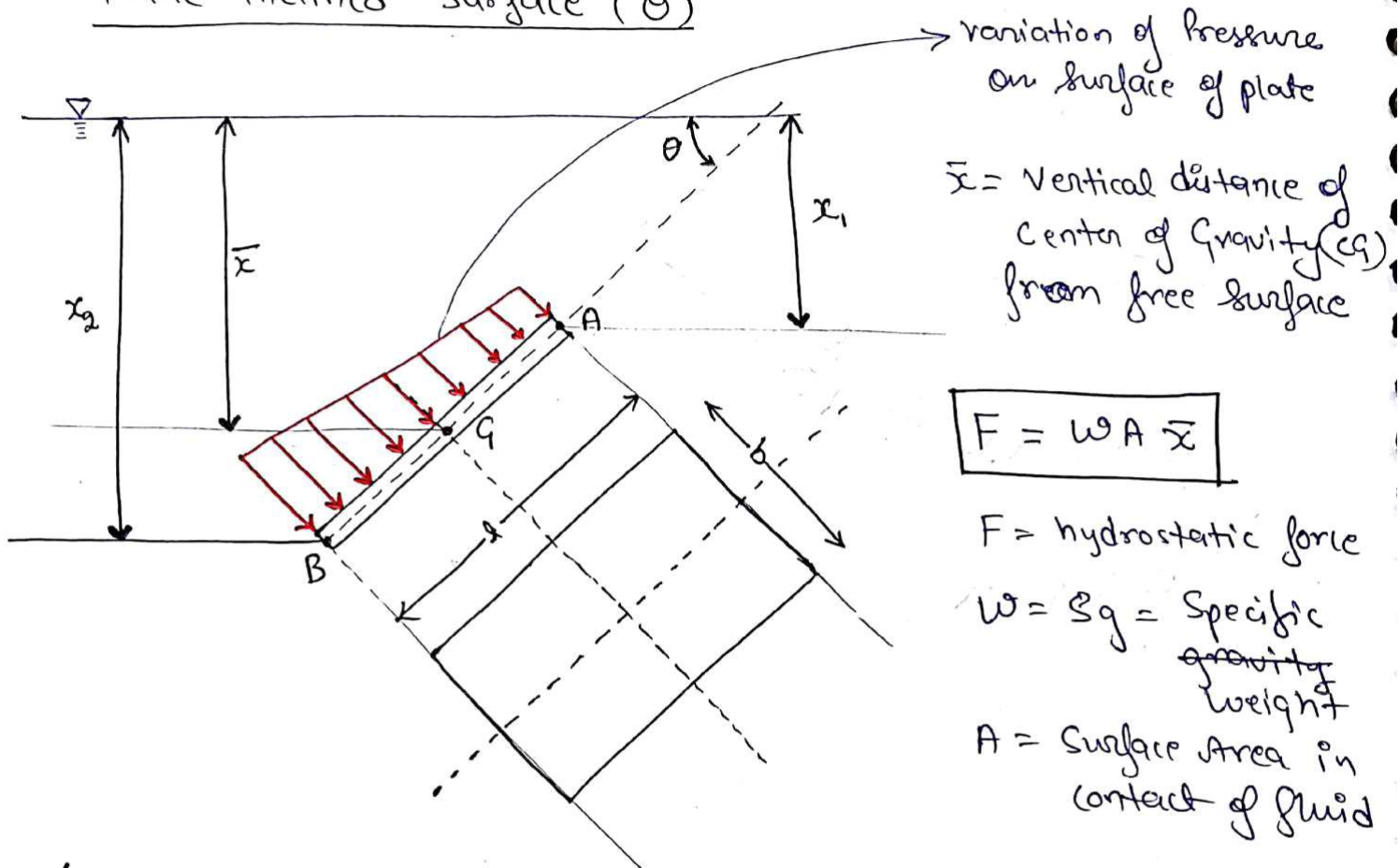
c) Bellow tube



## B) HYDROSTATIC FORCES

### a) hydrostatic forces on plane surfaces

#### 1. Plane inclined Surface ( $\theta$ )



$$F = \omega A \bar{x}$$

$F$  = hydrostatic force

$\omega = \rho g$  = Specific gravity weight

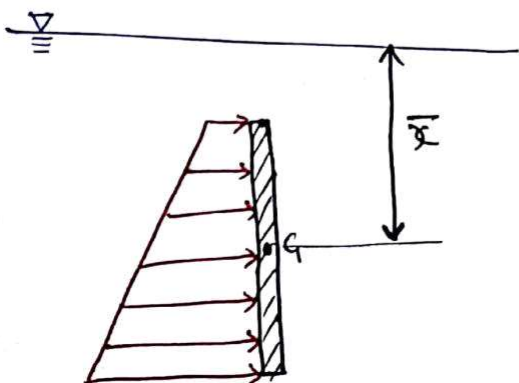
$A$  = Surface Area in contact of fluid

$$h = \frac{I_g \sin^2 \theta}{A \bar{x}} + \bar{x}$$

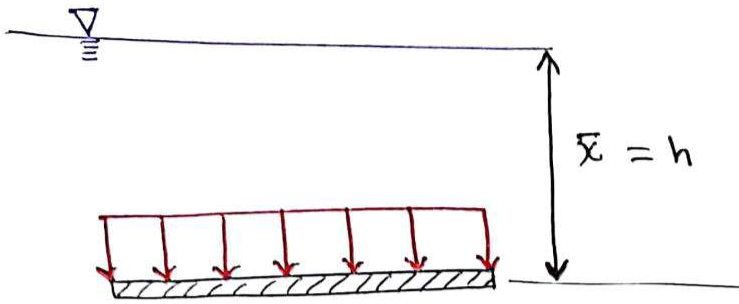
$h$  = Vertical distance of center of Pressure from free surface

$I_g$  = Moment of inertia @ Centroidal axis.

#### 2) Plane vertical Surface ( $\theta = 90^\circ$ )



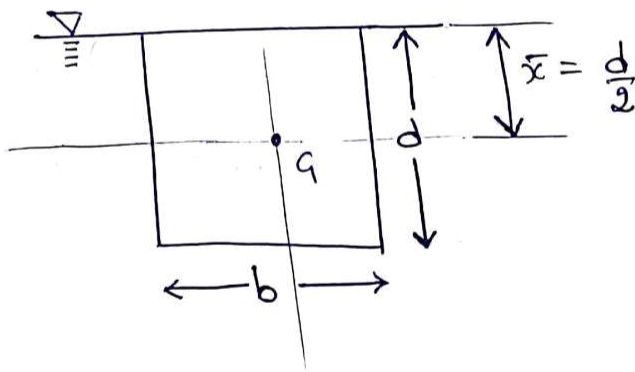
3) Plane horizontal Surface ( $\theta = 0^\circ$ )



- Note:
- The magnitude of hydrostatic force remains unchanged with angle of inclination.
  - The center of pressure which is the point of application of hydrostatic force always lies either below center of gravity or coincides with center of gravity.
  - Center of Pressure (C.P.) can never lie above center of gravity.

P-13  
Q1

A)

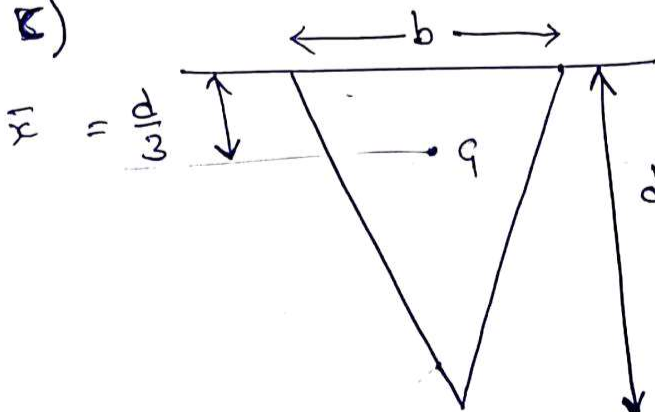


$$h = \bar{x} + \frac{I_{Gx} \sin^2 \theta}{A \bar{x}}$$

$$h = \frac{d}{2} + \frac{bd^3}{12 \times bd \times d} \times \frac{2}{d}$$

$$h = \frac{d}{2} + \frac{d}{6} = \frac{2d}{3}$$

B)



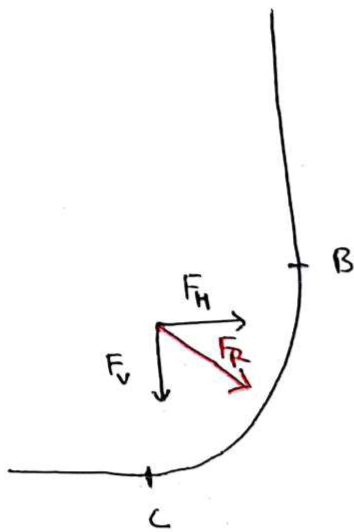
$$h = \bar{x} + \frac{I_{Gx} \sin^2 \theta}{A \bar{x}}$$

$$h = \frac{d}{3} + \frac{bd^3}{36} \times \frac{2}{bd} \times \frac{3}{d}$$

$$h = \frac{d}{3} + \frac{d}{6}$$

$$h = \frac{2d}{3}$$

## b) Hydrostatic forces on curved surfaces



BC = curved surface

$F_v$  = Vertical component of  $F_R$

$F_H$  = Horizontal component of  $F_R$

$F_R$  = Resultant force on curved surface.



### • Horizontal component of force on curved surface

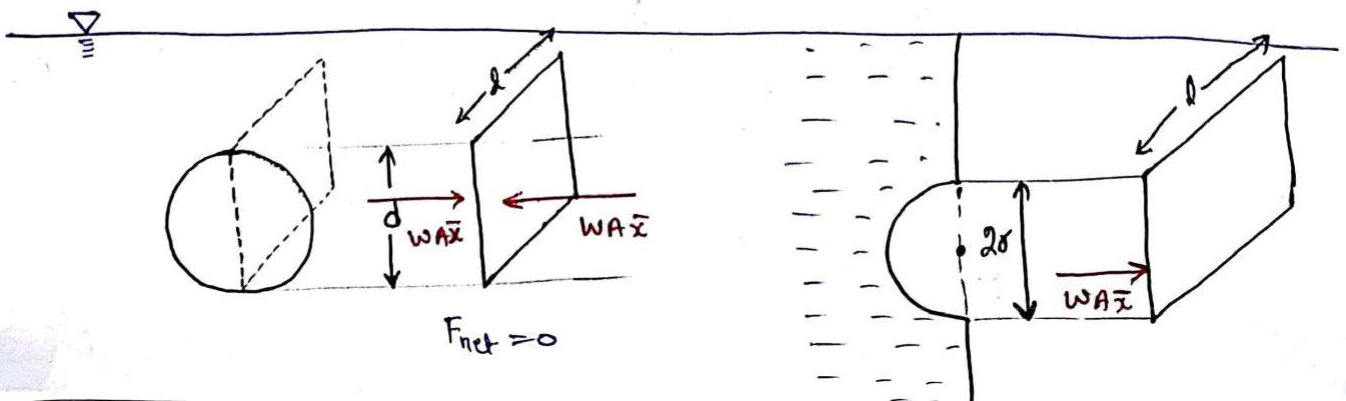
The horizontal component of force on a curved surface is equal to hydrostatic force on the vertical projection area and this force will act at center of pressure of the corresponding area.



### • Vertical component of force on curved surface

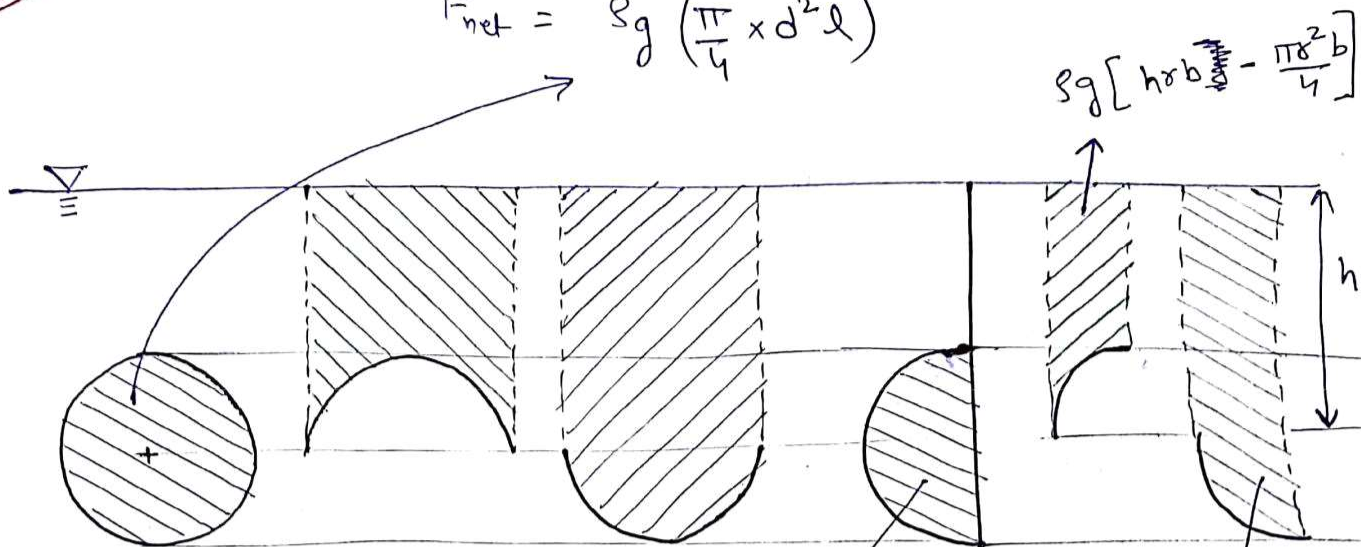
The vertical component of force on a curved surface is equal to weight of the fluid contained by the curved surface till the free surface and this force will act at the center of gravity of the corresponding weight.

Horizontal

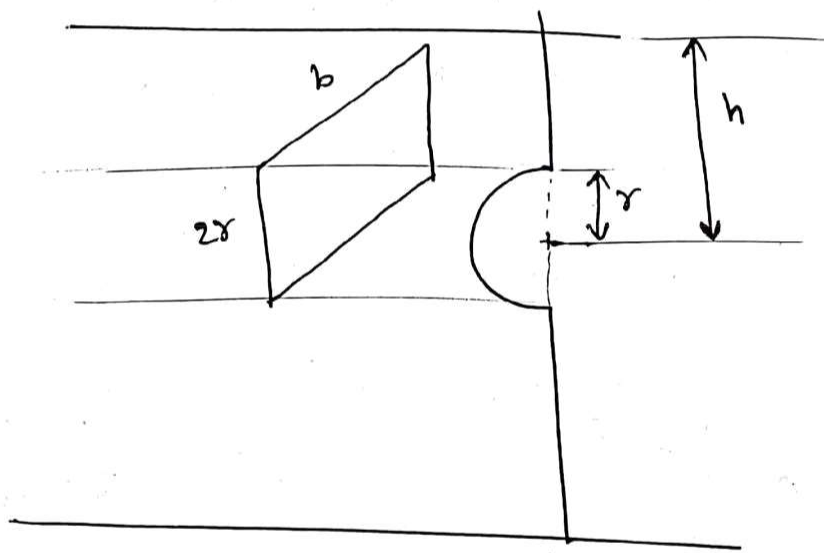


Vertical

$$F_{net} = \rho g \left( \frac{\pi}{4} \times d^2 l \right)$$



P-14  
Q6

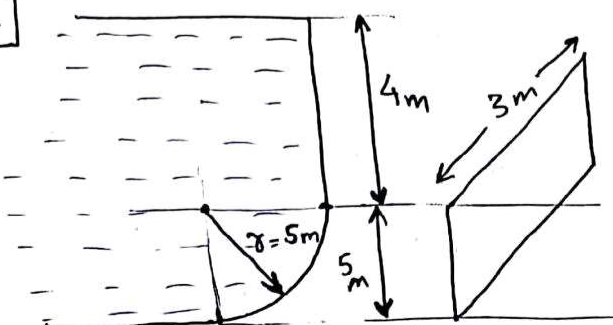


$$\begin{aligned} F_H &= \rho A \bar{x} \\ &= \rho \times 2r \times b \times h \\ &= 2 \rho r b h \end{aligned}$$

$$F_v = \rho g \times \left[ \frac{\pi r^2 b}{2} \right]$$

Note: The horizontal component on a cylinder completely submerged in fluid is zero whereas there will be a net vertical force on the cylinder.

P-14  
Q7



$$\begin{aligned} F_H &= \rho A \bar{x} = \rho \times 5 \times 3 \times 6.5 \\ &= 956.475 \text{ kN} \end{aligned}$$

$$\begin{aligned} F_v &= \rho g \times \left[ 4 \times 5 \times 3 + \frac{\pi}{4} (5)^2 \times 3 \right] \\ &= 1166.458 \text{ kN} \\ &= 1.166 \text{ MN} \end{aligned}$$

## c) Buoyancy & Floatation

- whenever a body is either partially or fully submerged in a fluid it experiences a net vertical force and this force is known as buoyancy force.
- According to Archimedes buoyancy force is equal to weight of the fluid displaced and this is known as Archimedes Principle.

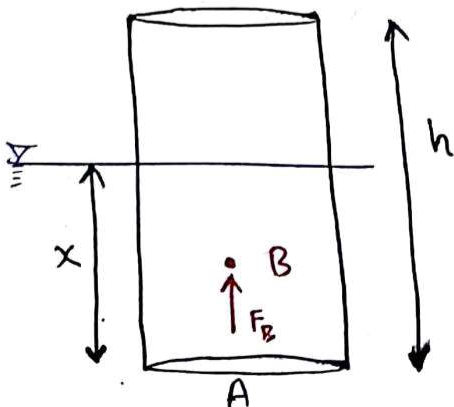
### Center of buoyancy (B)

Center of buoyancy is defined as the point of application of buoyancy force and this force will act at the centroid of volume of fluid displaced.

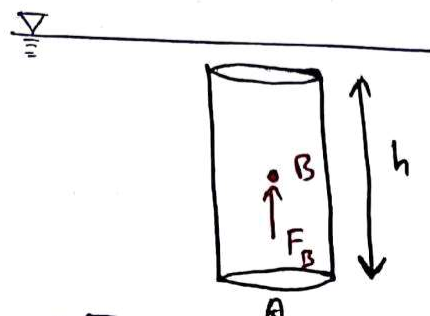
$$F_B = \text{wt. of fluid displaced}$$

$$F_B = \rho_f \cdot g \cdot V_{\text{fluid displaced}}$$

$$F_B = \rho_f \cdot g \cdot V_{\text{body submerged}}$$



$$\uparrow F_B = \rho_f \cdot g \cdot A \cdot x$$

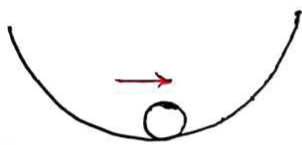


$$\uparrow F_B = \rho_f \cdot g \cdot A \cdot h$$

- Note:
- True weight of the body is measured in Air.
  - while measuring weight the body should be completely submerged.
  - The loss of weight in a fluid is equal to buoyancy force.

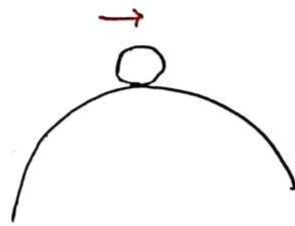
## Types of Equilibrium

1. Stable Equi.



Body will stop at its original position

2. unstable Equi.



Body will neither stop nor return to its original position

3. Neutral Equi

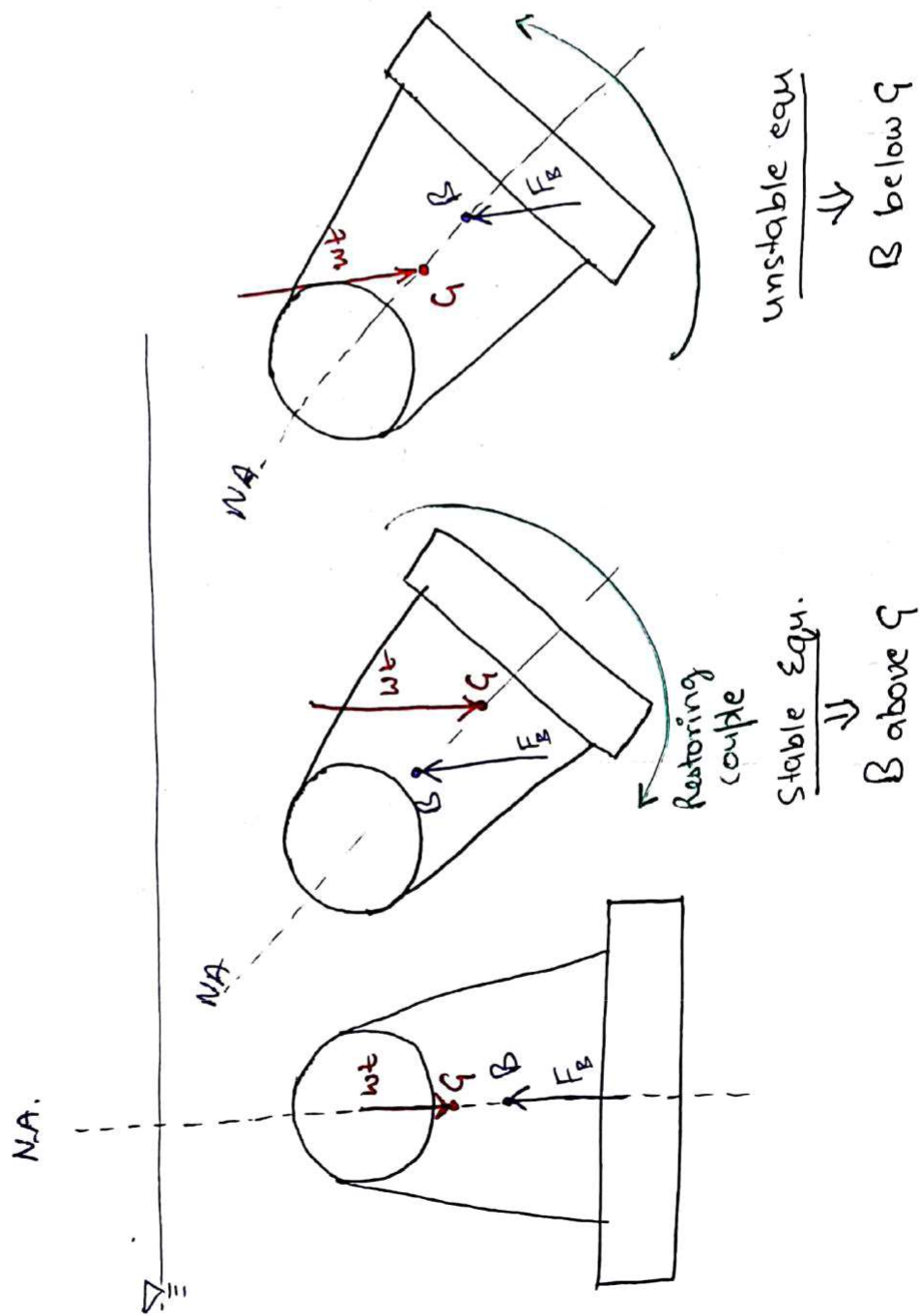


Body will stop but at some other position than original.

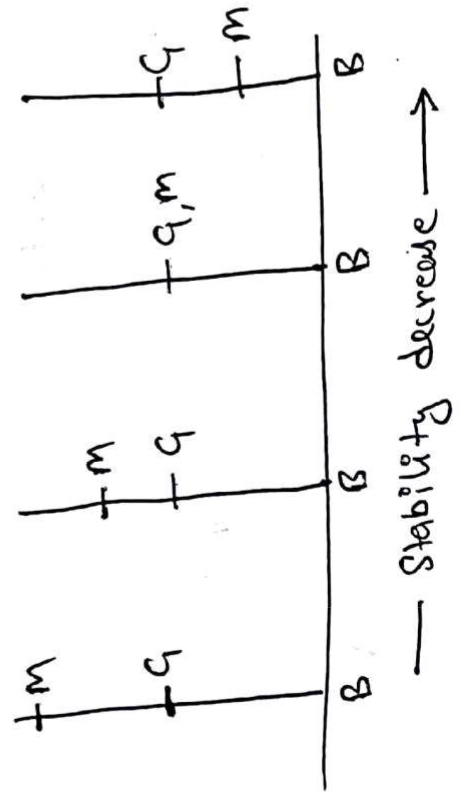
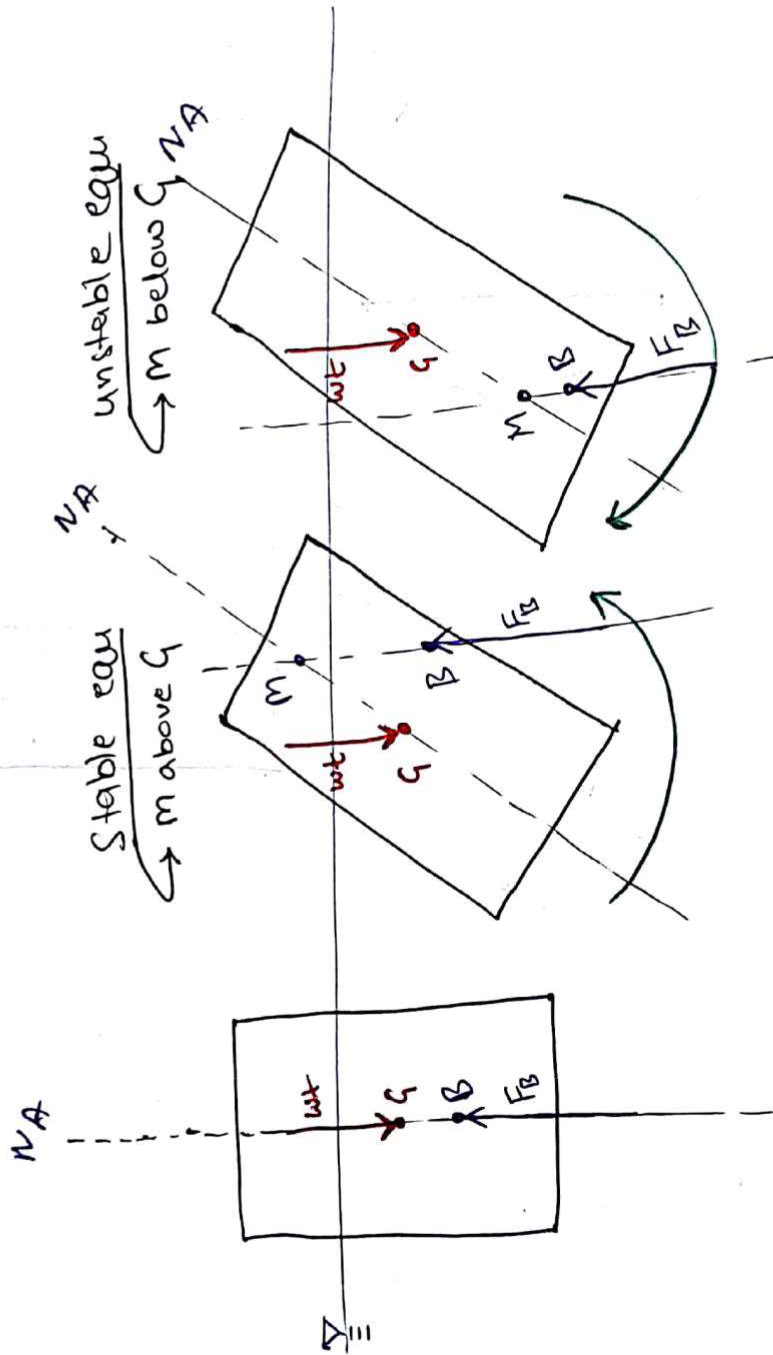
- Note:
- A floating body under vertical displacement is an example of stable equilibrium.
  - A floating body under horizontal displacement is an example of neutral equilibrium.



# Stability conditions of a completely submerged bodies under angular deflection



# Stability conditions of partially submerged bodies under angular deflection



## Equilibrium

### Stable

- $M$  above  $G$
- $Bm > Bg$
- $Gm = +ve$

### Unstable

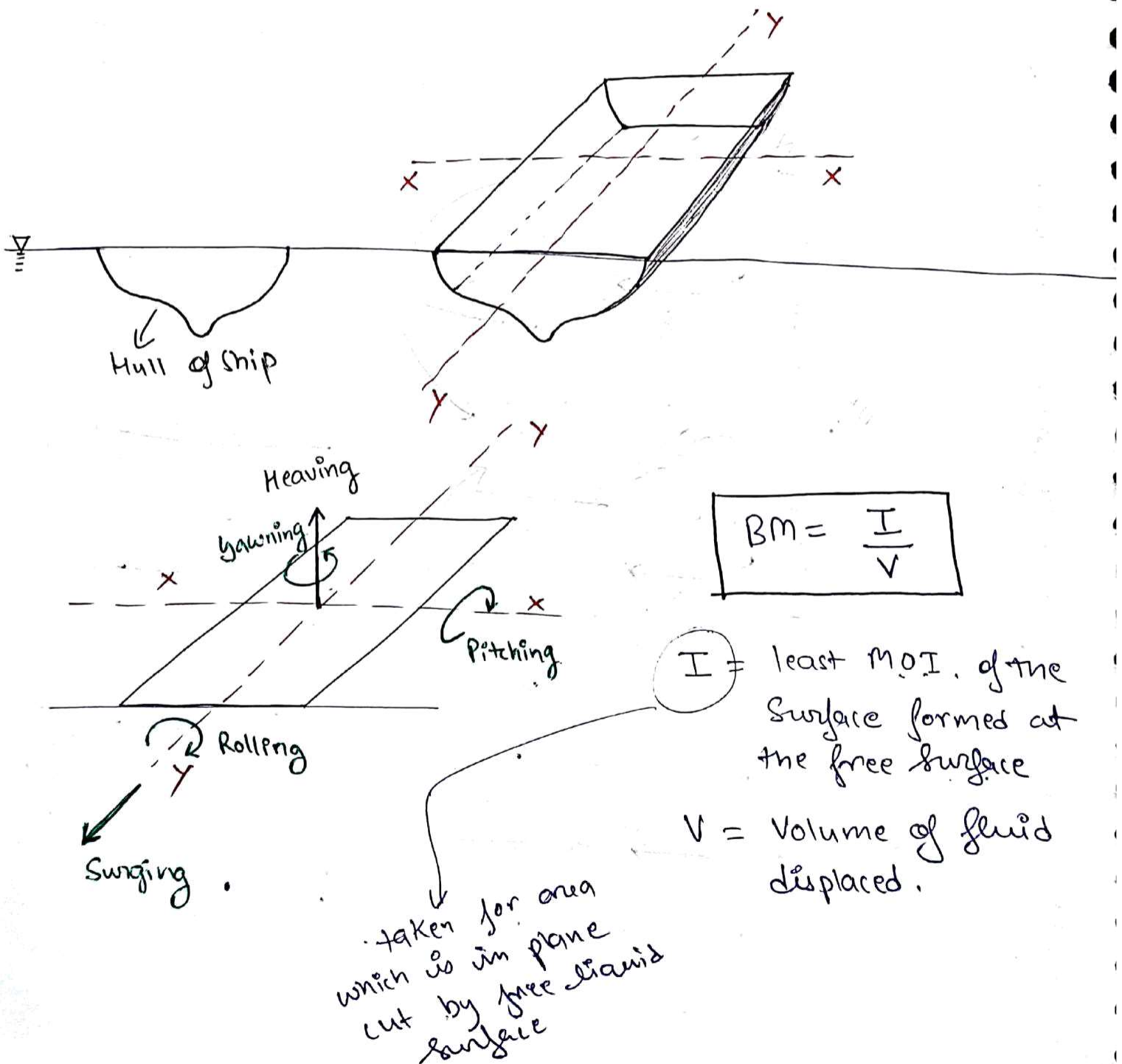
- $M$  below  $G$
- $Bm \leq Bg$
- $Gm = -ve$

# Calculation of metacentric height (Gm)

Metacenter (M) it is defined as the point of intersection of normal Axis and new line of Action of buoyancy force.

Metacentric Height (Gm) it is defined as the vertical distance b/w metacenter "M" and center of gravity "G"

- For stable equilibrium Gm should be positive.



$$I_{xx} = \frac{bl^3}{12} \quad ; \quad I_{yy} = \frac{lb^3}{12}$$

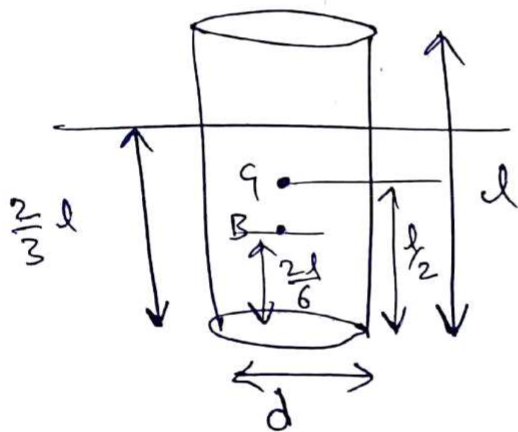
$$(BM)_{xx} = \frac{I_{xx}}{V} \quad ; \quad (BM)_{yy} = \frac{I_{yy}}{V}$$

$$(BM)_{xx} > (BM)_{yy}$$

$$(BM)_{\text{pitching}} > (BM)_{\text{rolling}}$$

Note: While designing boats or ships,  $(BM)_{\text{rolling}}$  must always be taken care of by designing by rolling condition, the metacentric height comes true then the boat will be stable in all the cases.

P-17  
Q7



$$\frac{I}{V} \geq \frac{l}{2} - \frac{l}{3} = \frac{l}{6}$$

$$\frac{\pi d^2 \times l \times 3}{16 \times \pi d^2 \times 2l} \geq \frac{l}{6}$$

$$d^2 \geq \frac{32l^2}{18}$$

$$\Leftrightarrow d^2 \geq \frac{16}{9} l^2$$

$$d \geq \frac{4}{3} l$$

P-18  
Q8

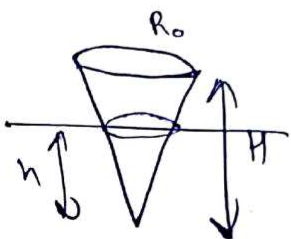
Note: Center of gravity of cone is  $\frac{3H}{4}$  from the apex

$$h = SH$$

$$r = \frac{hR}{H} = SR$$

$$I = \frac{\pi}{64} \times \frac{r^4}{16} \quad ; \quad V = \frac{1}{3} \pi r^2 h$$

$$\frac{I}{V} = \frac{\pi}{64} \times \frac{r^4}{16} \times \frac{3}{\pi r^2 h}$$



$$BG = \frac{3H}{4} - \frac{3SH}{4} = \frac{3H}{4}(1-S) < \frac{3}{16 \times 64} \times \frac{S^2 R^2}{SH}$$

## CHAPTER - 3

### FLUID KINEMATICS

- Now in fluid mechanics the fluid will start to flow and we will study only the motion of the fluid without considering the reason behind that motion (forces).
- According to ~~Eulerian~~ <sup>Lagrangian</sup> technique he considered a fluid particle and by tracing the path of fluid particle and studying the properties, the fluid flow analysis was done.
- Lagrangian technique was TDS and complex and hence it didn't gain popularity.
- According to Eulerian technique a  $x$ - $s/c$  was considered in a fluid flow and the fluid properties studied at this given  $x$ - $s/c$  of fluid flow was analysed. This technique became more famous due to its simplicity and hence it is used in fluid mechanics.

### Types of fluid flow

#### 1. Steady / unsteady flow

A steady flow is the one in which fluid properties are not changing w.r.t. but at a given  $x$ - $s/c$ .

$$\left( \frac{\partial v}{\partial t} \right)_{\text{at a } x\text{-}s/c} = 0 \quad ; \quad \left( \frac{\partial s}{\partial t} \right)_{\text{at } x\text{-}s/c} = 0$$

#### 2. Uniform / non-uniform flow

A uniform flow is one in which velocity of fluid is not changing w.r.t.  $x$ - $s/c$  but at a given interval of time.

$$\left[ \frac{\partial v}{\partial (x/s/c)} \right]_{\text{at given time}} = 0$$

## Examples

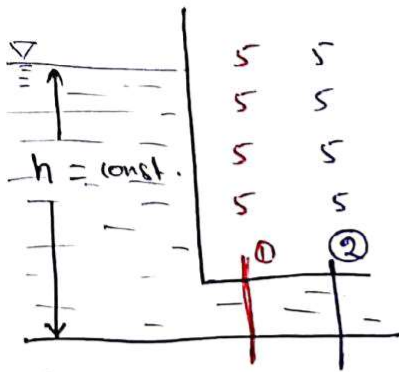


Fig: Steady uniform flow

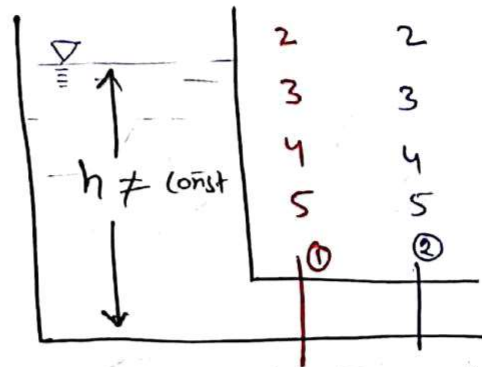


Fig: unsteady uniform flow

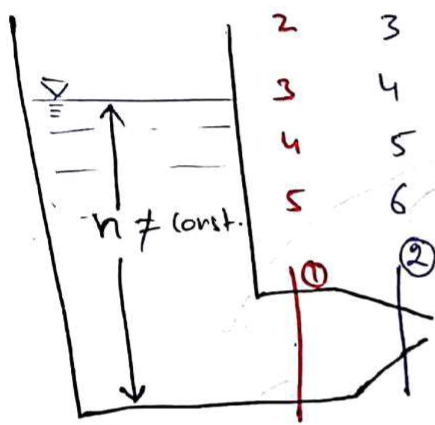


Fig: unsteady & non-uniform flow

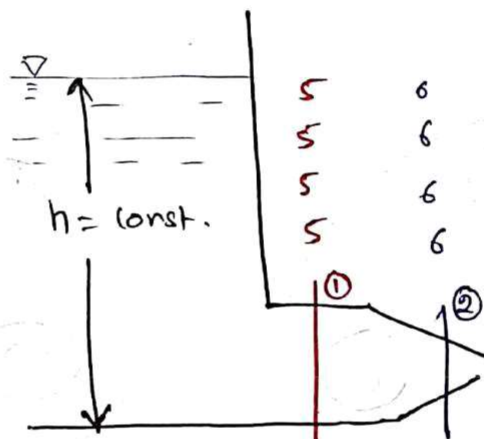
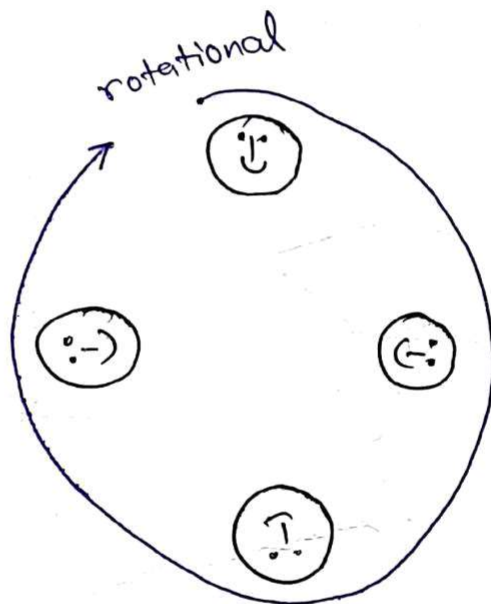
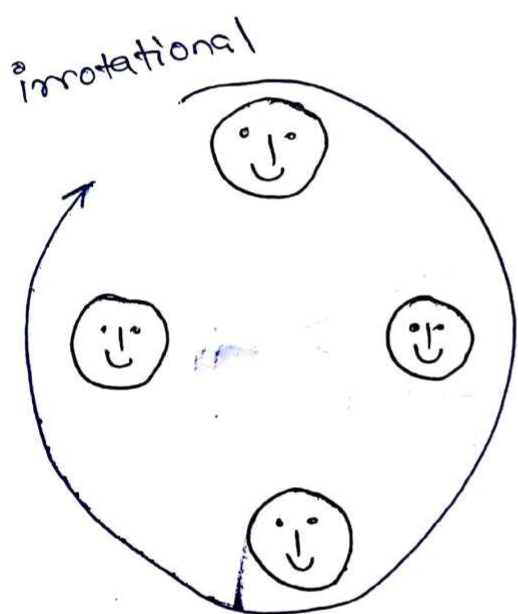
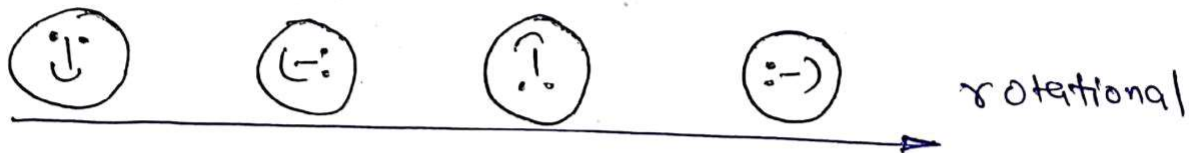


Fig: steady & non-uniform flow

### 3. Laminar and turbulent flow

- A laminar flow is the one in which fluid moves in the form of layers and there is no intermixing of fluid particles or molecular momentum transfer.  
ex:- flow of blood in veins.
- Laminar flow generally occurs at low velocity & high viscosity.
- A turbulent flow is the one in which there is huge order intermixing of fluid particles or molecular momentum transfer.
- Turbulent flow generally occurs at high velocity & low viscosity [ex:- flow of water in rivers]  
ex:- flow of exhaust gases through chimney.

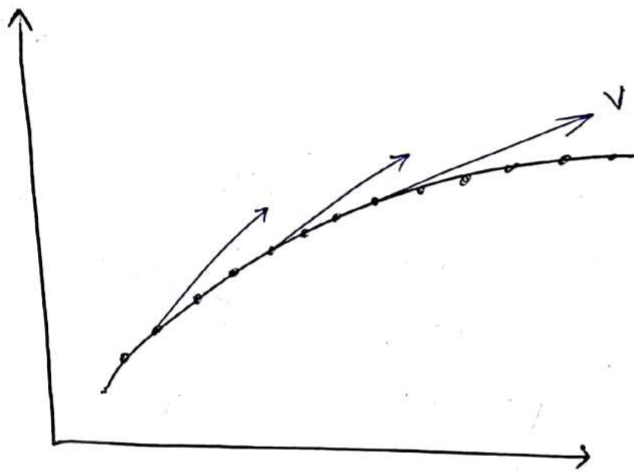
#### 4. Rotational & irrotational flow



- A rotational flow is the one in which the fluid particles rotate about their mass centers otherwise the flow is irrotational.
- For a fluid to be rotational a certain tangential shear is required to produce torque on the fluid particle, which is provided by viscosity and hence it can be concluded that non viscous fluids can never be rotational.

## Tools for studying fluid flow

1. Streamline it is an imaginary curve drawn in space such that tangent drawn to it at any point will give velocity of fluid flow. Streamline gives an instantaneous picture of the fluid flow. As there is no component of velocity across the stream line, the flow always takes place along the stream line. Two streamlines can never intersect with itself, neither a streamline can never intersect with itself.



at a given time

### Velocity of a fluid Partical

$$\vec{v} = u\vec{i} + v\vec{j} + w\vec{k}$$

where  $u, v, w$  are the components of velocity in  $x, y, z$  dir<sup>n</sup>.

$$v = \sqrt{u^2 + v^2 + w^2}$$

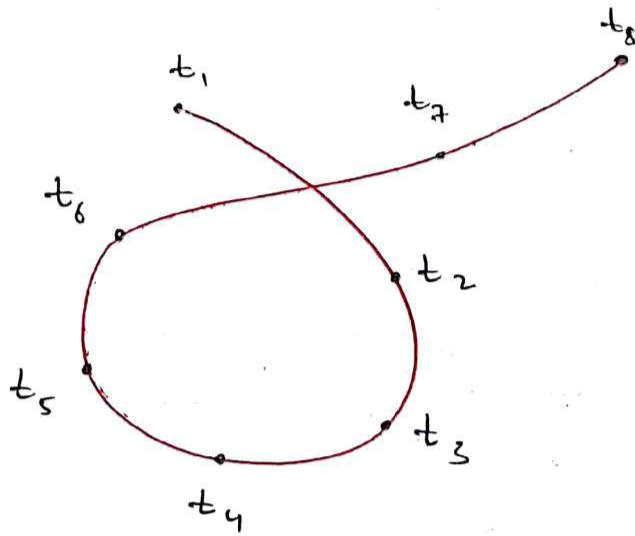
### Equation of streamline

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}} \rightarrow 3D$$

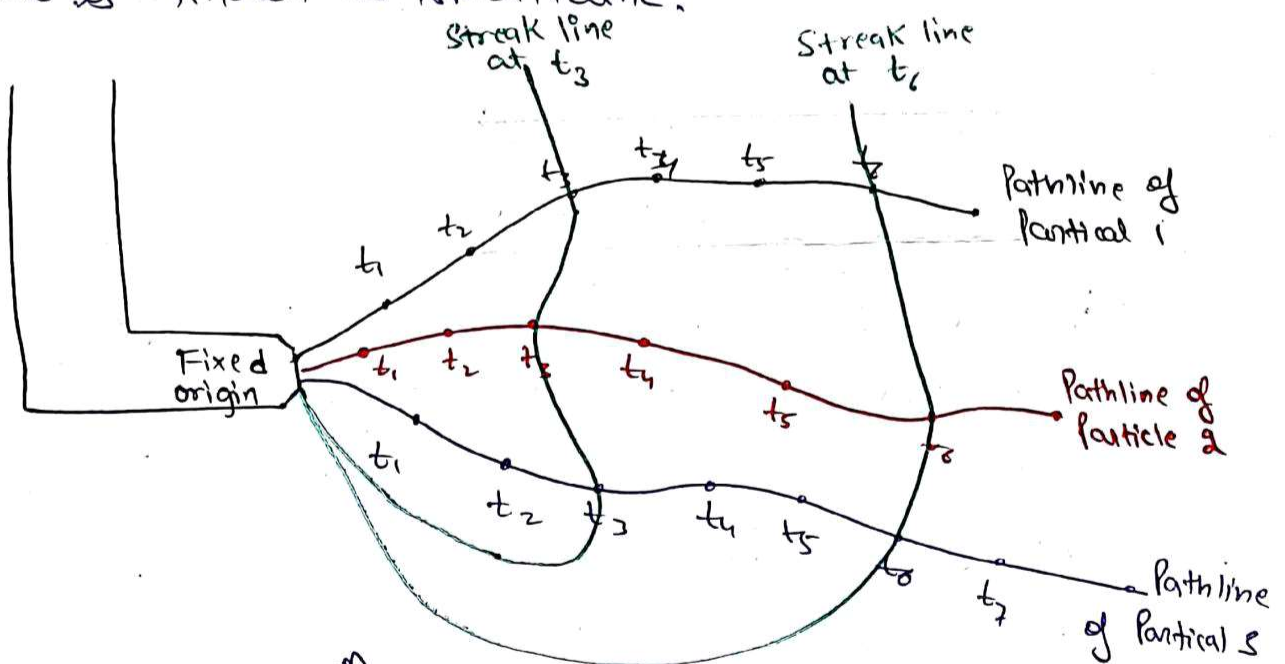
$$\boxed{\frac{dx}{u} = \frac{dy}{v}} \rightarrow 2D$$



2. Path line A path line is the line formed by joining the locations of a single fluid particle at different interval of time. A path line can intersect with itself.



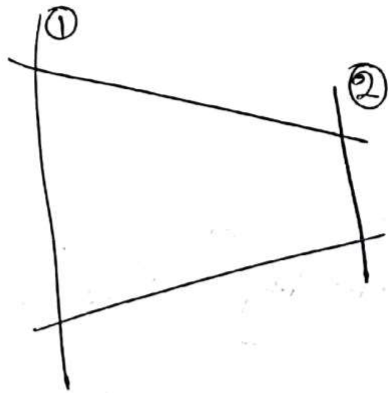
3. Streak line it is the locus of different fluid particles originating from a single fixed point at a given interval of time is known as streak line.



Note: • For a steady <sup>uniform</sup> flow stream line, streak line & path line will coincide whereas for an unsteady flow they will never coincide

# Continuity equation [conservation of mass]

- Steady, 1D  $\begin{cases} \dot{m}_{\text{entering}} = \dot{m}_{\text{leaving}} \\ E_{\text{entering}} = E_{\text{leaving}} \end{cases}$



$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

T&C

- 1) Steady
- 2) 1D

- for an incompressible fluid

$$A_1 V_1 = A_2 V_2$$

T&C

- 1) Steady
- 2) 1D
- 3) incompressible

- Discharge ( $Q$ )

$$Q = AV \text{ m}^3/\text{s}$$

$$Q = Q_1 = Q_2 = A_1 V_1 = A_2 V_2 = \text{constant}$$

T&C

- 1) Steady
- 2) 1D
- 3) incompressible

- $\rho AV = \text{const.}$

differentiating both side

$$\rho A dv + A v ds + v s dA = 0$$

dividing by  $\rho AV$

$$\frac{dv}{v} + \frac{dA}{A} + \frac{ds}{s} = 0$$

T&C

- 1) Steady
- 2) 1D

## Generalised continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$$

valid for all cases

$$\Rightarrow \frac{d\rho}{dt} + \rho(\text{div. } \vec{v}) = 0$$

$$\Rightarrow \boxed{\frac{d\rho}{dt} + \rho(\nabla \cdot \vec{v}) = 0}$$

vector form of generalised continuity equation.

$$\text{here, } \nabla \cdot \vec{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z}$$

Case-1 steady flow

$$\boxed{\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0}$$

Case-2 incompressible flow ( $\rho = \text{const.}$ )

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0} \rightarrow 3D$$

$$\Rightarrow \nabla \cdot \vec{v} = 0$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0} \rightarrow 2D$$

Note: • Every fluid flow must satisfy its continuity eqn<sup>n</sup> otherwise the flow is not possible.

# Acceleration of a fluid Particle

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

where,  $a_x, a_y, a_z$  are the components of acceleration of fluid particle in  $x, y, z$  dir<sup>n</sup>

$$\begin{aligned}
 a_x &= u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \\
 a_y &= u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \\
 a_z &= u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}
 \end{aligned}
 \Rightarrow \text{local component}$$

$\swarrow$  convective component

	local acc <sup>n</sup>	convective acc <sup>n</sup>
1. Steady & uniform	0	0
2. unsteady & uniform	exist	0
3. Steady & nonuniform	0	exist
4. unsteady & nonuniform	exist	exist

## Rotation of a fluid Particle

$$\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$

where,  $\omega_x, \omega_y, \omega_z$  are the components of rotation vector  $\vec{\omega}$  in  $x, y, z$  dir<sup>n</sup>

$$\boxed{\vec{\omega} = \frac{1}{2} \text{curl } \vec{V}} = \frac{1}{2} \nabla \times \vec{V} = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$\vec{\omega} = \frac{1}{2} \left[ \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right] \hat{i} - \frac{1}{2} \left[ \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} \right] \hat{j} + \frac{1}{2} \left[ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right] \hat{k}$$

$$\Rightarrow \begin{cases} \omega_x = \frac{1}{2} \left[ \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right] \\ \omega_y = \frac{1}{2} \left[ \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right] \\ \omega_z = \frac{1}{2} \left[ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right] \end{cases}$$

- if  $\omega_x = \omega_y = \omega_z = 0 \Rightarrow$  flow is irrotational
- Vorticity  $(\zeta) = 2 \vec{\omega} = \text{curl } \vec{V} \Rightarrow \boxed{\text{Vorticity} = \nabla \times \vec{V}} = \zeta$
- $\boxed{\text{Circulation} = \text{Vorticity} \times \text{Area}} = \Gamma$

• For 2D case

$$\text{if } \omega_z = 0 \Rightarrow \begin{aligned} \omega &= 0 \\ \text{Vorticity} &= 0 \\ \text{Circulation} &= 0 \end{aligned}$$

## Velocity Potential function ( $\phi$ )

it is a function of space such that the -ve derivative of this function w.r.t. a given direction will give component of velocity in that direction.

$$\boxed{-\frac{\partial \phi}{\partial x} = u}$$

$$\boxed{-\frac{\partial \phi}{\partial y} = v}$$

$$\boxed{-\frac{\partial \phi}{\partial z} = w}$$

• For 2D  $\rightarrow$  incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\text{LHS} = \frac{\partial}{\partial x} \left( -\frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( -\frac{\partial \phi}{\partial y} \right)$$

$$= -\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} = - \left[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right]$$

$\hookrightarrow$  Laplace eqn<sup>n</sup>

$\Rightarrow$  Case-1

if  $\phi$  satisfies Laplace  $\Rightarrow$  flow is possible

$\Rightarrow$  Case-2

if  $\phi$  does not satisfy Laplace  $\Rightarrow$  flow is not possible

$$w_z = \frac{1}{2} \left[ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right]$$

$$= \frac{1}{2} \left[ \frac{\partial}{\partial x} \left( -\frac{\partial \phi}{\partial y} \right) - \frac{\partial}{\partial y} \left( -\frac{\partial \phi}{\partial x} \right) \right]$$

$$= \frac{1}{2} \left( -\frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial x \partial y} \right)$$

$= 0 \Rightarrow$  if  $\phi$  exists  $\Rightarrow$  flow is irrotational

## Stream function ( $\psi$ ) [only defined for 2D]

- it is a function of space such that it satisfies continuity equation.

$$u = -\frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \psi}{\partial x}$$

- 2D incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\text{LHS} = \frac{\partial}{\partial x} \left( -\frac{\partial \psi}{\partial y} \right) + \frac{\partial}{\partial y} \left( \frac{\partial \psi}{\partial x} \right)$$

$$= -\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial x \partial y} = 0$$

$$\Rightarrow \omega_z = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$= \frac{1}{2} \left[ \frac{\partial}{\partial x} \left( \frac{\partial \psi}{\partial x} \right) - \frac{\partial}{\partial y} \left( -\frac{\partial \psi}{\partial y} \right) \right] = \frac{1}{2} \left[ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right]$$

Laplace  
↑↑

\*\*

Case-1: if  $\psi$  satisfy Laplace  $\rightarrow$  flow is irrotational

\*\*  
Case-2: if  $\psi$  does not satisfy Laplace  $\rightarrow$  flow is rotational

- Note:
- A line along which stream func<sup>n</sup>  $\psi$  is constant is known as streamline
  - A line along which velocity Potential func<sup>n</sup> ( $\phi$ ) is const. is known as equipotential line.

• The grid formed by joining the series of equipotential lines and stream lines is known as flow net.

✱ Equipotential lines & stream lines are orthogonal to each other.

• The difference b/w any two streamlines will give discharge per unit length. (discharge =  $|\psi_1 - \psi_2|$ )

Slope of stream function

$$\frac{dx}{u} = \frac{dy}{v}$$

$v dx - u dy = 0 \rightarrow$  eq<sup>n</sup> of stream line

$$u = -\frac{\partial \psi}{\partial y} \quad v = \frac{\partial \psi}{\partial x}$$

$$\frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy = 0$$

•  $\psi = f(x, y) \Rightarrow d\psi = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy$

$\Rightarrow d\psi = 0 \Rightarrow \psi = \text{constant}$

$\Rightarrow 0 = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy$

$\Rightarrow \frac{dy}{dx} = \frac{-\frac{\partial \psi}{\partial x}}{\frac{\partial \psi}{\partial y}} = \frac{-v}{u} = \frac{v}{-u}$

$\Rightarrow \left( \frac{dy}{dx} \right)_{\text{stream line}} = \frac{v}{-u} \quad \text{--- ①}$



## Slope of Equipotential lines

•  $\phi = f(x, y)$

$$d\phi = \frac{\partial \phi}{\partial x} \cdot dx + \frac{\partial \phi}{\partial y} \cdot dy$$

Equipotential  $\Rightarrow d\phi = 0 \Rightarrow \phi = \text{constant}$

$$0 = \frac{\partial \phi}{\partial x} \cdot dx + \frac{\partial \phi}{\partial y} \cdot dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\frac{\partial \phi}{\partial x}}{\frac{\partial \phi}{\partial y}} = -\frac{y}{x}$$

$$\Rightarrow \boxed{\left(\frac{dy}{dx}\right)_{\text{Equipotential}} = -\frac{y}{x}} \quad \text{--- (2)}$$

• from (1) & (2)

we can say equipotential lines & streamlines are  $\perp$ .

## Cauchy - Reiman Eqn

$$\boxed{\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}}$$

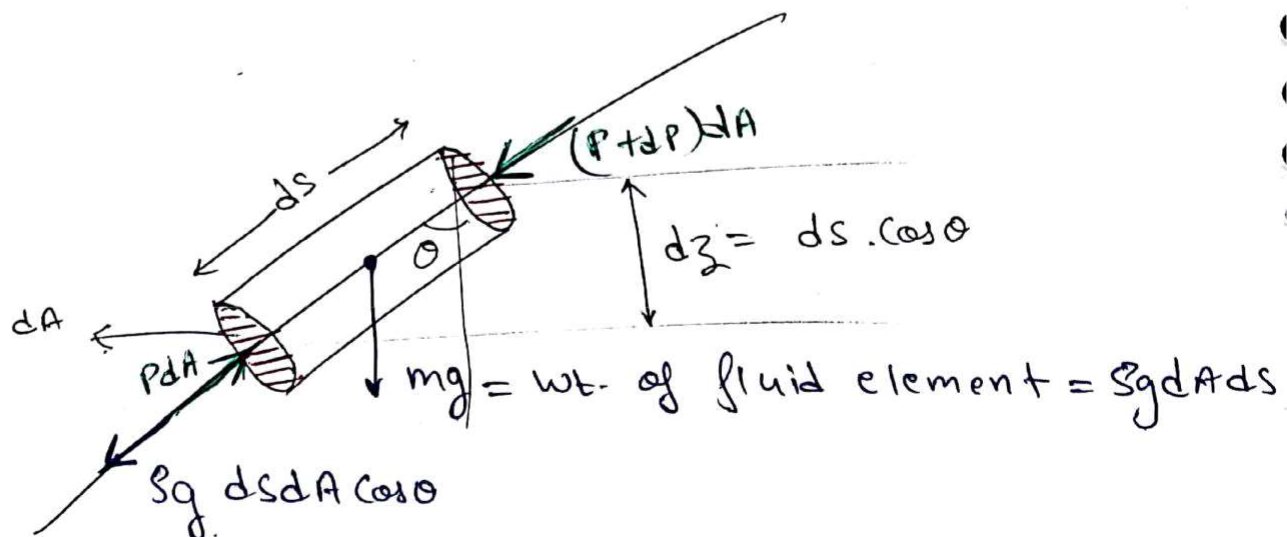
## CHAPTER - 4

### FLUID DYNAMICS

- Now in fluid dynamics we will consider the various forces behind the fluid flow.
- According to Navier-Stokes he considered pressure force  $F_p$ , gravity force  $F_g$ , viscous force  $F_v$ , and derived the Navier-Stokes equation.
- Euler further assumed the fluid to be non-viscous and by considering the pressure and gravity force, he derived the Euler's equation.
- Then by further assuming the fluid to be steady & incompressible Bernoulli derived his famous equation Bernoulli's equation, which became popular as conservation of energy equation.

#### Euler's Equation [ $F_p + F_g$ ]

Assumption: 1) flow is non-viscous



$$m = \frac{wt}{g} = \rho dA ds$$

$$a = a_x = u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}$$

$$a = v \frac{dv}{ds} + \frac{\partial v}{\partial t}$$

$$\Sigma F = m \times a$$

$$\cancel{P dA} - \cancel{P dA} - dP dA - \rho g dA ds \cos \theta = \rho dA ds \left[ v \frac{dv}{ds} + \frac{\partial v}{\partial t} \right]$$

$$-dP dA + \rho g dA ds \cos \theta = \cancel{\rho dA ds} \left( v \frac{dv}{ds} + \frac{\partial v}{\partial t} \right)$$

$$\Rightarrow \boxed{dP + \rho g dz + \rho ds \left( v \frac{dv}{ds} + \frac{\partial v}{\partial t} \right) = 0} \quad \text{Euler's eqn}$$

Assumption: 2) flow is steady  $\left[ \frac{\partial v}{\partial t} = 0 \right]$

$$dP + \rho g dz + \cancel{\rho ds} \cdot v \frac{dv}{\cancel{ds}} = 0$$

$$dP + \rho g dz + \rho v dv$$

Dividing by  $\rho$

$$\frac{dP}{\rho} + g dz + v dv = 0$$

Assumption: 3) considering incompressible fluid & integrating

$$\int \frac{dP}{\rho} + \int g dz + \int v dv = 0$$

$$\boxed{\frac{P}{\rho} + gz + \frac{v^2}{2} = \text{Const.}}$$

$$\frac{\text{Joules}}{\text{Kg}}$$

Dividing by "g"

$$\boxed{\frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{constant}} \quad \underline{\text{meters}}$$

↳ classical Bernoulli's equation

## Bernoulli's Theorem

- Bernoulli's theorem states that for a non-viscous steady & incompressible flow along a streamline the summation of kinetic energy, P.E., pressure energy is a constant and such an equation is known as Bernoulli's equation.
- Classical Bernoulli's eqn<sup>n</sup> is also known as mechanical energy conservation eqn<sup>n</sup> whereas S.F.E.E. is known as total energy conservation equation.

$$\frac{P}{\rho} + \frac{v^2}{2g} + z = \text{const.}$$

↓                      ↓                      ↓

Pressure head      Kinetic head      Potential head

⇒ Classical Bernoulli's eqn<sup>n</sup>  
[mechanical energy conservation eqn<sup>n</sup>]

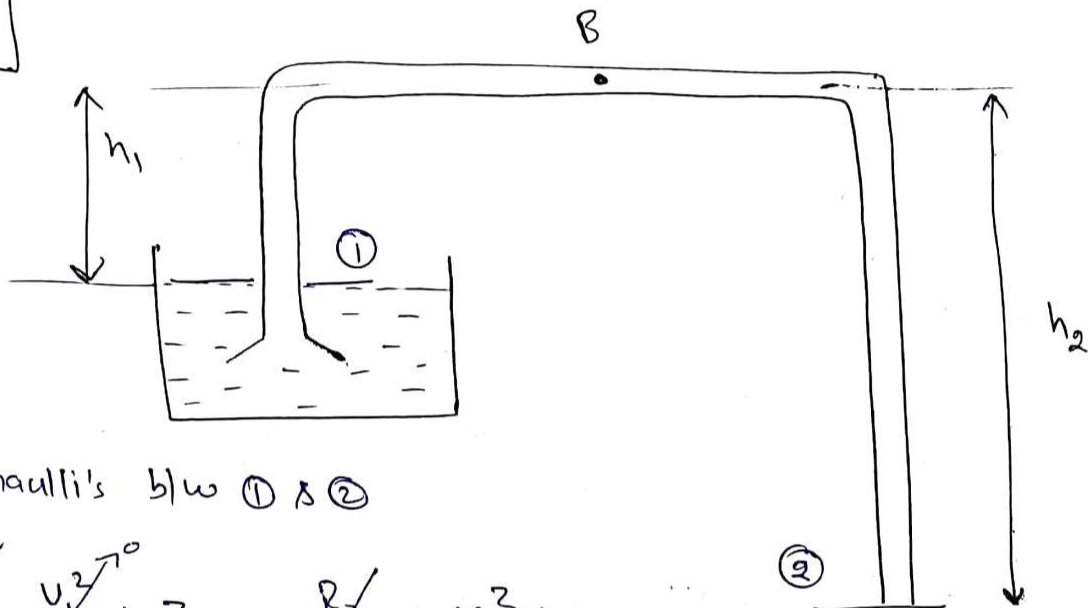
- Assumptions :
- 1) Non viscous
  - 2) Steady flow
  - 3) incompressible
  - 4) ideal fluid
  - 5) irrotational

$$\frac{P}{\rho} + z = \text{Piezometric head}$$

Ideal  $\Rightarrow \frac{P_1}{\rho} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + Z_2$

Real  $\Rightarrow \frac{P_1}{\rho} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + Z_2 + \text{head losses}$

P-27  
Q2



Bernoulli's b/w ① & ②

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + Z_2$$

$$h_2 - h_1 = \frac{V_2^2}{2g}$$

$$\Rightarrow V_2 = \sqrt{2g(h_2 - h_1)}$$

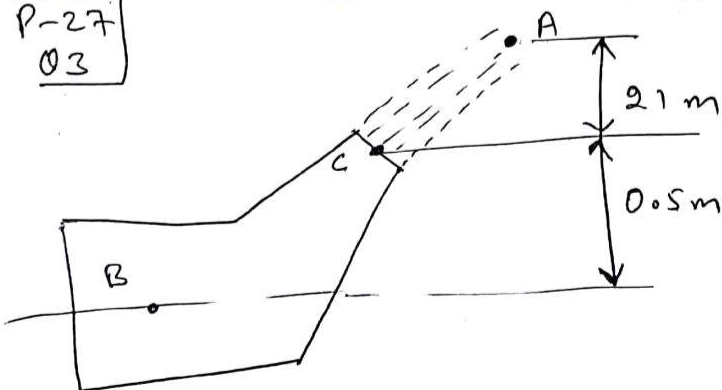
$$A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{A_2}{A_1} V_2$$

very small

continuity b/w B & C

P-27  
Q3



$$A_B V_B = A_C V_C \Rightarrow V_B = V_C$$

B. eqn b/w A & C

$$\frac{P_C}{\rho} + \frac{V_C^2}{2g} + Z_C = \frac{P_A}{\rho} + \frac{V_A^2}{2g} + Z_A$$

$$\frac{V_C^2}{2g} = \frac{(18)^2}{2g} + 21$$

$$\Rightarrow V_C = 27.1 \text{ m/s}$$

Continuity b/w B & C

$$A_B v_B = A_C v_C$$

$$\frac{\pi}{4} \times (225)^2 \times v_B = \frac{\pi}{4} \times (075)^2 \times 27.1$$

$$v_B = 3.01 \text{ m/s}$$

B. eqn b/w B & C

$$\frac{P_B}{\rho} + \frac{v_B^2}{2g} + z_B = \frac{P_C}{\rho} + \frac{v_C^2}{2g} + z_C$$

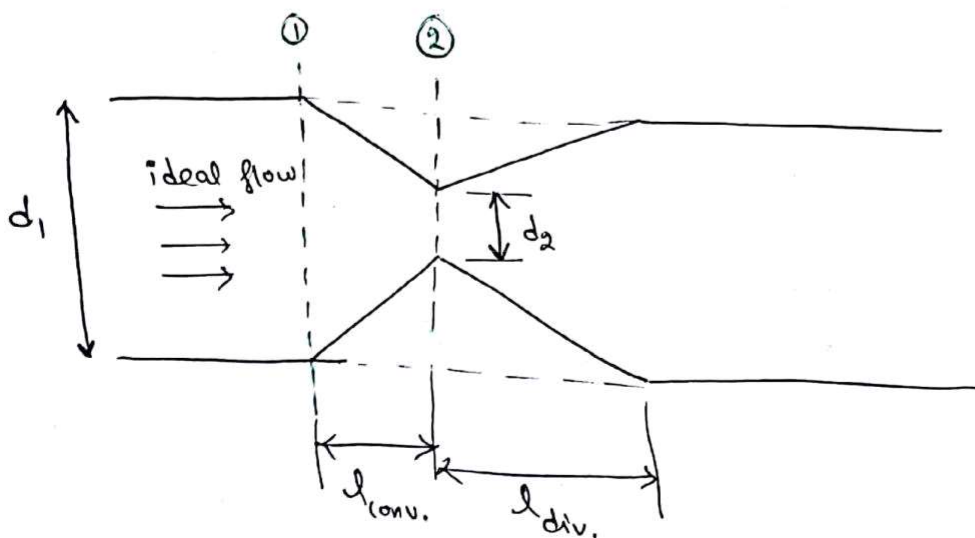
$$\frac{P_B}{9810} + \frac{(3.01)^2}{2g} = \frac{(27.1)^2}{2g} + 0.5$$

$$\Rightarrow P_B = 367.579 \text{ kPa}$$

## Applications of Bernoulli's equation

### 1) Venturimeter

it is a converging diverging device that is used to measure discharge



B- eqn<sup>n</sup> b/w ① & ②

$$\frac{P_1}{\omega} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\omega} + \frac{V_2^2}{2g} + z_2$$

for horizontal pipe ( $z_1 = z_2$ )

$$\frac{P_1}{\omega} + \frac{V_1^2}{2g} = \frac{P_2}{\omega} + \frac{V_2^2}{2g}$$

$$h \leftarrow \left( \frac{P_1 - P_2}{\omega} \right) = \frac{V_2^2 - V_1^2}{2g} \Rightarrow V_2^2 - V_1^2 = 2gh$$

$$\Rightarrow h = \frac{P_1 - P_2}{\omega} \longrightarrow \text{for horizontal pipe} \\ \text{[i.e. Pressure head difference]}$$

$$h = \left( \frac{P_1}{\omega} + z_1 \right) - \left( \frac{P_2}{\omega} + z_2 \right) \longrightarrow \text{for inclined pipe} \\ \text{[i.e. Piezometric head difference]}$$

$\Rightarrow$  for incompressible fluid

$$A_1 V_1 = A_2 V_2 = Q_1 = Q_2 = Q$$

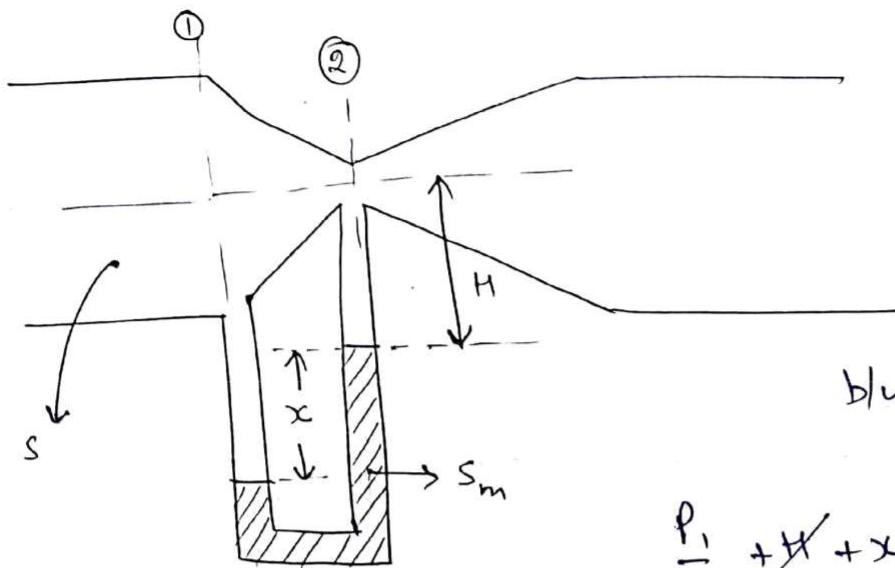
$$\Rightarrow V_2 = \frac{Q}{A_2} ; V_1 = \frac{Q}{A_1}$$

$$\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2} = 2gh$$

$$\Rightarrow Q^2 \left[ \frac{A_1^2 - A_2^2}{A_1^2 A_2^2} \right] = 2gh$$

$$\Rightarrow \boxed{Q_{\text{theo.}} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}}$$

# Calculation of "h"

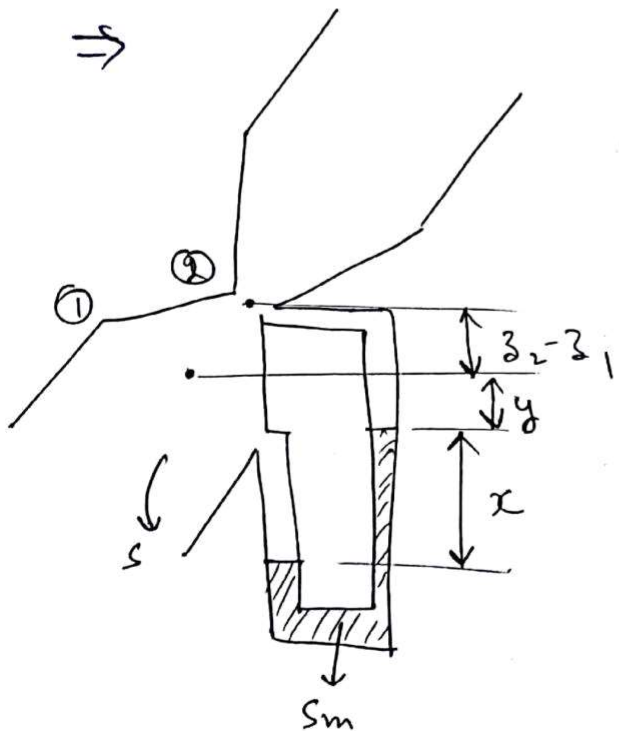


b/w ① & ②

$$\frac{P_1}{\rho} + H + x - \frac{S_m}{S}x - H = \frac{P_2}{\rho}$$

$$\Rightarrow \frac{P_1}{\rho} - \frac{P_2}{\rho} = x \left[ \frac{S_m}{S} - 1 \right]$$

$$\Rightarrow \boxed{h = x \left[ \frac{S_m}{S} - 1 \right]} \Rightarrow \text{for horizontal pipe}$$



$$\frac{P_1}{\rho} + y + x - \frac{S_m}{S}x - y - z_2 + z_1 = \frac{P_2}{\rho}$$

$$\left( \frac{P_1}{\rho} + z_1 \right) - \left( \frac{P_2}{\rho} + z_2 \right) = h = x \left[ \frac{S_m}{S} - 1 \right]$$

$$\Rightarrow \boxed{h = x \left[ \frac{S_m}{S} - 1 \right]}$$

Note:- For inverted type u-tube manometer

$$\boxed{h = x \left( 1 - \frac{S_m}{S} \right)}$$



## Coefficient discharge ( $C_d$ )

It is defined as ratio of actual discharge to theoretical discharge.

- 0.94 to .98
- losses are less
- Accuracy is high
- expansive

$$C_d = \frac{Q_{\text{actual}}}{Q_{\text{theoretical}}}$$

$$\Rightarrow Q_{\text{actual}} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh} \quad \text{--- (1)}$$

$$\Rightarrow \frac{P_1}{\omega} + \frac{V_1^2}{2g} = \frac{P_2}{\omega} + \frac{V_2^2}{2g} + h_e$$

$$\frac{P_1}{\omega} - \frac{P_2}{\omega} - h_e = \frac{V_2^2 - V_1^2}{2g}$$

$$(h - h_e) = \frac{V_2^2 - V_1^2}{2g} \Rightarrow Q_{\text{act.}} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2g(h - h_e)} \quad \text{--- (2)}$$

Comparing (1) & (2)

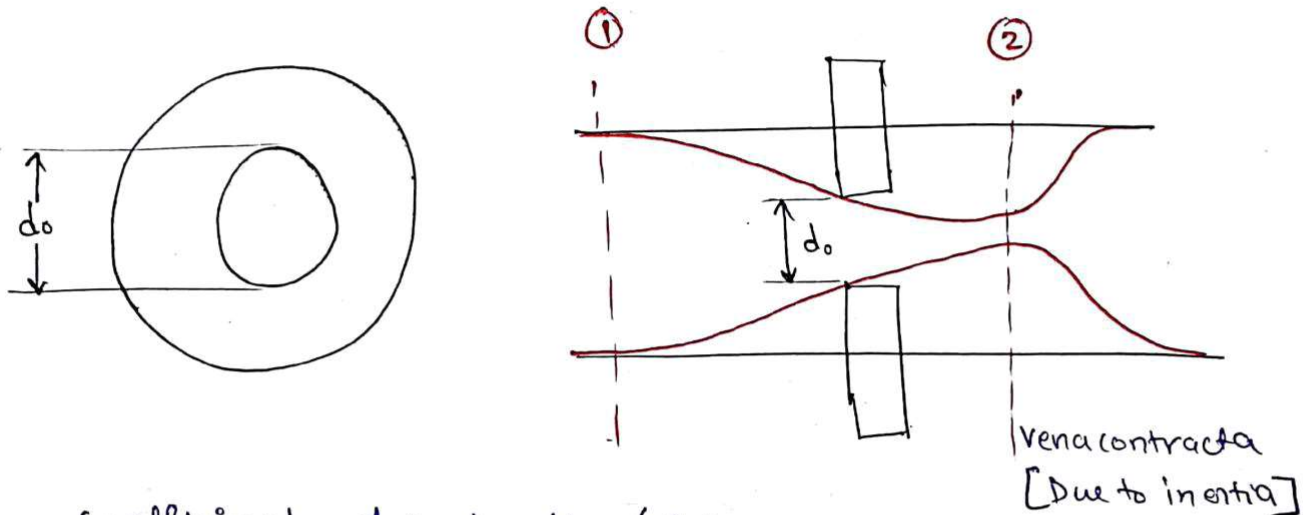
$$C_d = \sqrt{\frac{(h - h_e)}{h}}$$

here,  $h_e$  = head loss

$$h = x \left( \frac{S_m}{S} - 1 \right)$$

## 2) Orifice meter

- Orifice meter is used to measure the discharge of fluid flow and it is the cheapest device to measure discharge.
- The losses are very high in case of orificemeter and that is why the value of  $C_d$  is low.
- $C_d$  of an orifice meter lies b/w 0.68 to 0.74



Coefficient of contraction ( $C_c$ )

$$C_c = \frac{\text{Actual minimum Area}}{\text{Theoretical mini. Area}} = \frac{A_2}{A_0}$$

$$\Rightarrow A_2 = C_c A_0$$

Continuity eqn<sup>n</sup> b/w ① & ②

$$A_1 V_1 = A_2 V_2 \implies V_1 = \frac{A_2 V_2}{A_1} = \frac{C_c A_0 V_2}{A_1}$$

B- eqn<sup>n</sup> b/w ① & ②

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{P_2}{\rho} + \frac{V_2^2}{2g}$$

$$h = \frac{P_1 - P_2}{\rho} = \frac{V_2^2 - V_1^2}{2g}$$

$$\boxed{V_2^2 - V_1^2 = 2gh}$$

$$V_2^2 - \frac{C_c^2 A_0^2 V_2^2}{A_1^2} = 2gh$$

$$V_2^2 = \frac{\sqrt{2gh}}{\sqrt{1 - \frac{C_c^2 A_0^2}{A_1^2}}}$$

$$Q = A_2 V_2$$

$$= \frac{C_c A_0 \sqrt{2gh}}{\sqrt{1 - \frac{C_c^2 A_0^2}{A_1^2}}} \times \frac{\sqrt{1 - \frac{A_0^2}{A_1^2}}}{\sqrt{1 - \frac{A_0^2}{A_1^2}}} \Rightarrow C_d$$

$$Q = C_d \cdot \frac{A_1 A_0}{\sqrt{A_1^2 - A_0^2}} \times \sqrt{2gh}$$

$A_0$  = Area of orifice

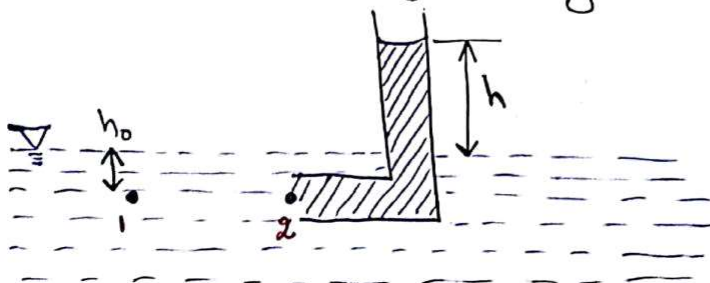
$A_1$  = Area of Pipe.

- The losses in case of orifice meter are high bcz the convergence of area is sudden and hence leading to losses
- The losses in case of ventury meter are less bcz the convergence of area is gradual and slow and hence losses are less.

### 3) Pitot tube

- A Pitot tube is a 90° bend glass tube that is used to measure ~~of~~ velocity of fluid flow.

Case-1 measurement of velocity in open channels



$$P_1 = \rho g h_0$$

$$h_0 = \frac{P_1}{\rho g}$$

$$V_2 = 0 \quad \left[ \begin{array}{l} \text{fluid isentropically} \\ \text{Brought to rest} \end{array} \right]$$

$$P_2 = \rho g (h + h_0)$$

$$h + h_0 = \frac{P_2}{\rho g}$$

Applying B-eqn b/w ① & ②

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{P_2}{\rho} + \frac{V_2^2}{2g}$$

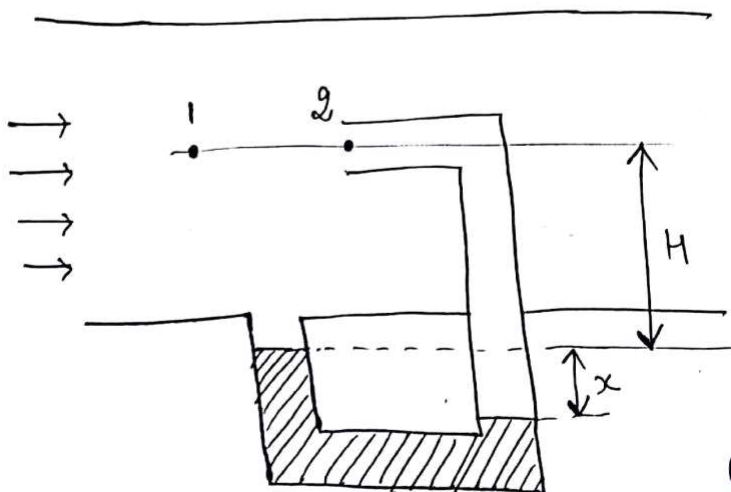
$$\boxed{\frac{P_1}{\rho}} + \boxed{\frac{V_1^2}{2g}} = \boxed{\frac{P_2}{\rho}}$$

↓
↓
↓  
 Static head      Dynamic head      Stagnation Head

$$\frac{V_1^2}{2g} = \frac{P_2}{\rho} - \frac{P_1}{\rho} = h + \cancel{h_0} - \cancel{h_0}$$

$$\Rightarrow \boxed{V_1 = \sqrt{2gh}}^{***} = \sqrt{2g (\text{Stagnation Head} - \text{Static Head})}$$

### Case-2 Velocity in Pipes



Applying manometry

$$\frac{P_1}{\rho} + H + \frac{S_m}{S} x - x - H = \frac{P_2}{\rho}$$

$$\frac{P_2 - P_1}{\rho} = x \left[ \frac{S_m}{S} - 1 \right]$$

Applying B-eqn b/w ① & ②

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{P_2}{\rho} + \frac{V_2^2}{2g}$$

# CHAPTER - 5

## LAMINAR FLOW [viscous incompressible flow]

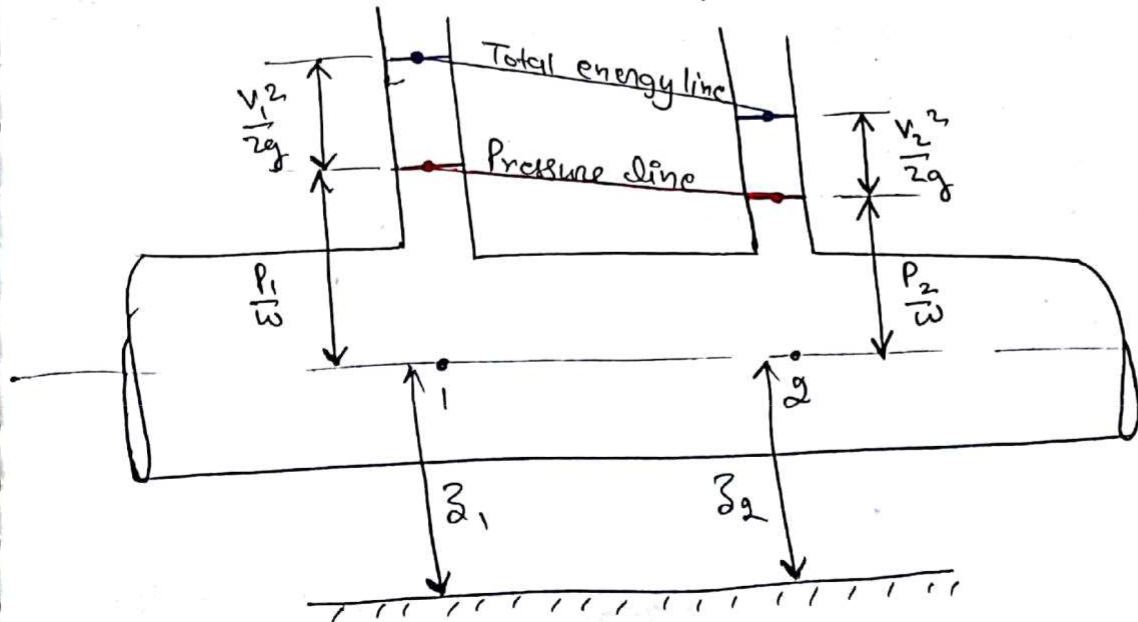
• Reynold's no. =  $\frac{\rho V D}{\mu} = \frac{V D}{\nu}$

D = characteristic length

= diameter  $\longrightarrow$  for pipe [ $D = \frac{4A}{P}$  for non circular x-s/c]

= length  $\longrightarrow$  for plate

<u>Pipe</u>	<u>Plate</u>
$Re < 2000$ laminar	$Re < 5 \times 10^5$ laminar
$2000 < Re < 4000$ Transient	$Re > 5 \times 10^5$ turbulent
$Re > 4000$ turbulent	[transient is small & neglected]
<u>open channels</u>	<u>soil</u>
$Re < 500$ laminar	$Re < 1$ laminar
$500 < Re < 1000$ transient	$1 < Re < 2$ Transient
$Re > 1000$ turbulent	$Re > 2$ turbulent



Continuity eqn

$$A_1 V_1 = A_2 V_2$$

$V_1 = V_2$

⇒ B-equation for horizontal pipe

$$\frac{P_1}{\rho} + \cancel{\frac{v_1^2}{2g}} = \frac{P_2}{\rho} + \cancel{\frac{v_2^2}{2g}} + h_e$$

$$\frac{P_1 - P_2}{\rho} = h_e$$

⇒ B-equation for inclined pipe

$$\frac{P_1}{\rho} + \cancel{\frac{v_1^2}{2g}} + z_1 = \frac{P_2}{\rho} + \cancel{\frac{v_2^2}{2g}} + z_2 + h_e$$

$$\left(\frac{P_1}{\rho} + z_1\right) - \left(\frac{P_2}{\rho} + z_2\right) = h_e$$

⇒ Darcy Weisbach equation

$$h_e = \frac{f l v^2}{2gD}$$

valid for all

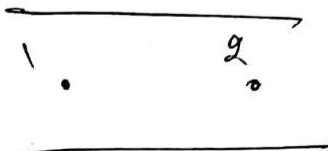
- laminar
- turbulent
- horizontal
- inclined

here,  $f$  = friction factor

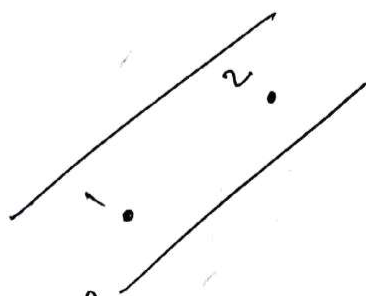
$$f = 4f' ; f' = \text{Fanning factor}$$

⇒ Direction of fluid flow

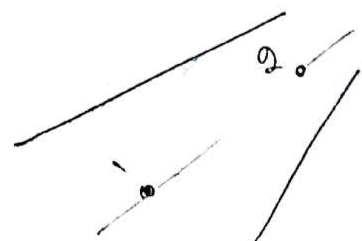
The fluid always flows from higher energy to lower energy.



Pressure energy will decide the flow.



Piezometric head will decide the flow



Total energy will decide the flow

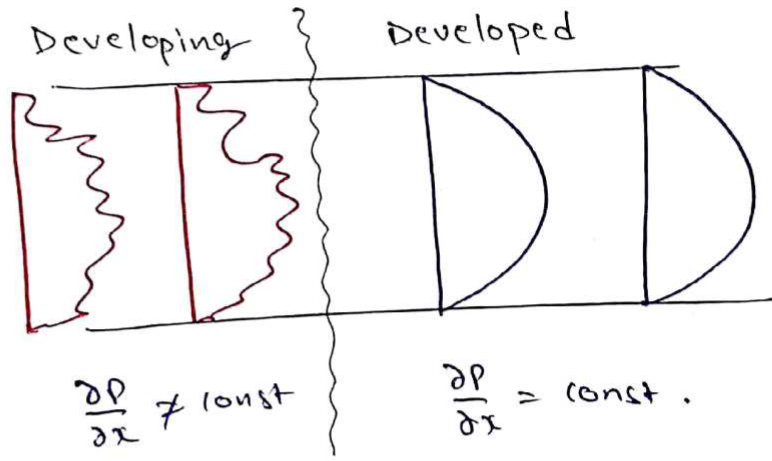
# Haigen - Poiseuille flow [laminar flow in pipe]

Assumptions: 1) steady flow

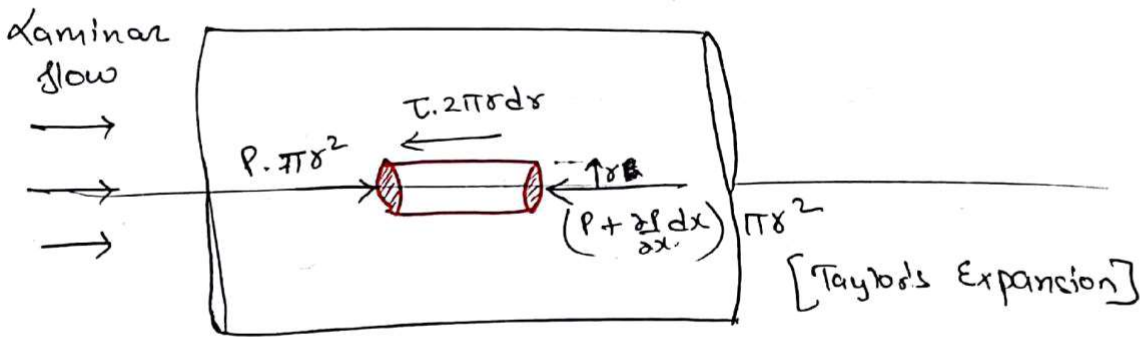
2) flow is fully developed

velocity profile is const. w.r.t. length

$\frac{\partial p}{\partial x} = \text{const.}$



## 1) shear stress distribution

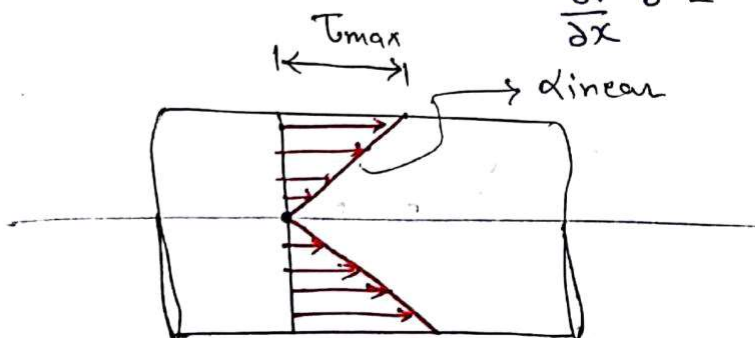


$\Sigma F = ma \Rightarrow \Sigma F = 0$

~~$P \pi r^2 - (P + \frac{\partial p}{\partial x} dx) \pi r^2 = \tau 2 \pi r dx$~~

$-\frac{\partial p}{\partial x} r = \tau \times 2$

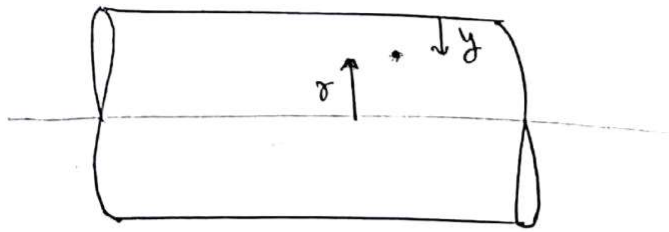
$\tau = -\frac{\partial p}{\partial x} \times \frac{r}{2}$



$\tau_{max} = -\frac{\partial p}{\partial x} \times \frac{R}{2}$

Shear stress distribution in Pipe [Haigen-Poiseuille flow]

2) velocity distribution in laminar flow through pipe



$$r + y = R$$

$$dr + dy = 0$$

$$dr = -dy$$

$$\tau = \mu \frac{du}{dy} = -\mu \frac{du}{dr} = \text{equation (1)}$$

$$\mu \frac{du}{dr} = \frac{\partial p}{\partial x} \cdot \frac{r}{2}$$

$$du = \frac{1}{2\mu} \left( \frac{\partial p}{\partial x} \right) \cdot r \, dr \Rightarrow u = \frac{1}{4\mu} \left( \frac{\partial p}{\partial x} \right) r^2 + c$$

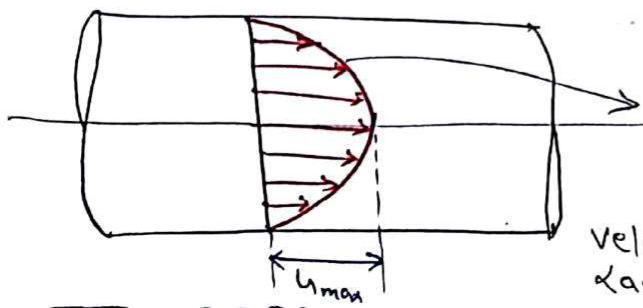
at  $r = R$   
 $u = 0$

$$0 = \frac{1}{4\mu} \left( \frac{\partial p}{\partial x} \right) R^2 + c \Rightarrow c = -\frac{1}{4\mu} \left( \frac{\partial p}{\partial x} \right) R^2$$

$$u = \frac{1}{4\mu} \left( \frac{\partial p}{\partial x} \right) r^2 - \frac{1}{4\mu} \left( \frac{\partial p}{\partial x} \right) R^2$$

$$u = -\frac{1}{4\mu} \left( \frac{\partial p}{\partial x} \right) R^2 \left( 1 - \frac{r^2}{R^2} \right) \quad \text{--- (2)}$$

$$u_{\max} = -\frac{1}{4\mu} \left( \frac{\partial p}{\partial x} \right) R^2 \Rightarrow \frac{u}{u_{\max}} = 1 - \frac{r^2}{R^2}$$



Parabolic (local velocity)

Velocity Distribution in laminar flow through pipes



### 3) Discharge ( $Q$ )

$$dQ = u dA$$

$$= \frac{-1}{4\mu} \left( \frac{\partial P}{\partial x} \right) R^2 \left( 1 - \frac{r^2}{R^2} \right) 2\pi r dr$$

$$= u_{max} \times 2\pi \int_0^R \left( r - \frac{r^3}{R^2} \right) dr$$

$$= u_{max} \times 2\pi \left[ \frac{r^2}{2} - \frac{r^4}{4R^2} \right]_0^R$$

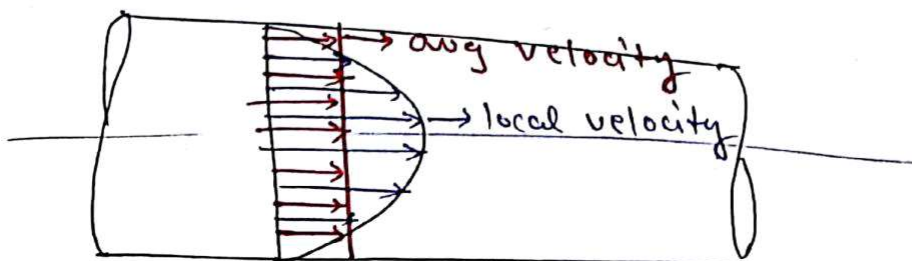
$$= u_{max} \cdot 2\pi \cdot \left( \frac{R^2}{2} - \frac{R^2}{4} \right) = u_{max} \cdot \frac{\pi R^2}{2}$$

$$Q = u_{max} \times \frac{A}{2} = V_{avg} \times A$$

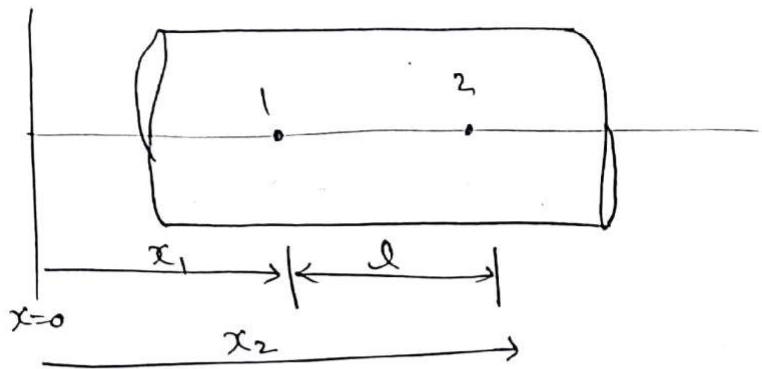
$$\Rightarrow V_{avg} = V = \frac{u_{max}}{2} \Rightarrow \text{for laminar flow through pipe}$$

$$\Rightarrow Q = V \cdot A$$

Note: in a pipe flow the velocity varies in  $y$ -dir<sup>n</sup> whereas the pressure varies in  $x$ -dir<sup>n</sup>. And hence a pipe flow is a two dimensional flow but a pipe flow is converted from 2D to 1D flow by the concept of average velocity. The variable velocity " $u$ " are replaced by a constant average velocity that gives the same result and hence 2D is converted to 1D.



#### 4) Pressure drop in laminar flow through pipe



$$V = \frac{u_{\max}}{2}$$

$$u_{\max} = 2V$$

$$-\frac{1}{4\mu} \left( \frac{\partial P}{\partial x} \right) R^2 = 2V$$

$$-\partial P = \frac{8\mu V (\partial x)}{R^2}$$

$$\Rightarrow \boxed{P_1 - P_2 = \frac{8\mu V l}{R^2}}$$

$$\Rightarrow \boxed{P_1 - P_2 = \frac{32\mu V l}{D^2}}$$

Valid for only horizontal pipe [laminar]

$$P_1 - P_2 = \rho g h_f$$

$$\rho g \times \frac{f l V^2}{2gD} = \frac{32\mu V l}{D^2} \Rightarrow$$

$$\boxed{f = \frac{64}{Re}}$$

Valid for both horizontal & inclined pipe [laminar]

#### 5) Shear velocity ( $V^*$ )

$$\tau = -\frac{\partial P}{\partial x} \cdot \frac{r}{2}$$

at  $r=R$ ,  $\tau = \tau_0$

$$\tau_0 = -\frac{\partial P}{\partial x} \cdot \frac{R}{2}$$

$$\tau_0 = -\frac{(P_2 - P_1)}{(x_2 - x_1)} \cdot \frac{R}{2}$$

$$\tau_0 = \frac{(P_1 - P_2)}{l} \times \frac{D}{4}$$

$$\tau_0 = \rho g \times \frac{f l V^2}{2gD} \times \frac{D}{4} \times \frac{1}{l}$$

$$\boxed{\tau_0 = \frac{\rho f V^2}{8}}$$

valid for both  $\rightarrow$  laminar  $\rightarrow$  turbulent

$$\boxed{V^* = V \cdot \sqrt{\frac{f}{8}} = \sqrt{\frac{\tau_0}{\rho}}}$$

$$\text{at } y=0 ; u=0$$

$$0 = \frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) 0 + c_1 \cdot 0 + c_2 \Rightarrow c_2 = 0$$

$$\text{at } y=B ; u=0$$

$$0 = \frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) B^2 + c_1 B$$

$$\Rightarrow c_1 = -\frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) B$$

$$\Rightarrow u = \frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) y^2 - \frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) B y$$

$$u = \frac{1}{2\mu} \left( -\frac{\partial P}{\partial x} \right) (B y - y^2)$$

$$\text{at } y = \frac{B}{2} ; u = u_{\max}$$

$$u_{\max} = \frac{1}{2\mu} \left( -\frac{\partial P}{\partial x} \right) \left( B \times \frac{B}{2} - \frac{B^2}{4} \right)$$

$$u_{\max} = \frac{1}{2\mu} \left( -\frac{\partial P}{\partial x} \right) \frac{B^2}{4} \Rightarrow u_{\max} = \frac{1}{8\mu} \left( -\frac{\partial P}{\partial x} \right) B^2$$

2. Discharge (Q)

$$dQ = u \, dy \times 1$$

$$dQ = \frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) (B y - y^2) \, dy$$

$$Q = \frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) \times \int_0^B (B y - y^2) \, dy$$

$$Q = \frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) \times \left( \frac{B y^2}{2} - \frac{y^3}{3} \right) \Big|_0^B$$

$$Q = \frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) \times \frac{B^3}{6} \Rightarrow Q = \frac{1}{12\mu} \left( \frac{\partial P}{\partial x} \right) B^3$$

## Average velocity

$$Q = AV$$

$$-\frac{1}{12\mu} \left(\frac{\partial p}{\partial x}\right) B^3 = B \times 1 \times v$$

$$v = -\frac{1}{12\mu} \left(\frac{\partial p}{\partial x}\right) B^2 \quad ; \quad u_{\max} = -\frac{1}{8\mu} \left(\frac{\partial p}{\partial x}\right) B^2$$

$$\Rightarrow \boxed{v_{\text{avg}} = \frac{2}{3} u_{\max}}$$

## 3. Pressure difference

$$v = -\frac{1}{12\mu} \left(\frac{\partial p}{\partial x}\right) B^2$$

$$-\partial p = \frac{12\mu v}{-B^2} \Delta x$$

$$\boxed{p_1 - p_2 = \frac{12\mu v L}{B^2}}$$

## 4. Shear stress distribution

$$\tau = \mu \frac{\partial u}{\partial y} = \mu \frac{\partial}{\partial y} \left[ -\frac{1}{2\mu} \left(\frac{\partial p}{\partial x}\right) (By - y^2) \right]$$

$$\tau = -\frac{1}{2} \left(\frac{\partial p}{\partial x}\right) (B - 2y)$$

$$\Rightarrow \boxed{\tau = \left(-\frac{\partial p}{\partial x}\right) \left(\frac{B}{2} - y\right)}$$

$$\boxed{\tau_{\max} = \left(-\frac{\partial p}{\partial x}\right) \cdot \frac{B}{2}}$$

# CHAPTER - 6

## FLOW THROUGH PIPES

- There are two types of losses in pipe i.e. major loss and minor loss.
- Major loss is due to friction whereas minor losses are divided into five types of losses that includes sudden contraction loss, sudden expansion loss, Exit loss, entry loss, bend loss.

### Major losses

(friction  $\Rightarrow$  95%)

#### 1. Darcy weisbach equation

$$h_f = \frac{f l v^2}{2gD}$$

$$Q = Av \Rightarrow v = \frac{4Q}{\pi D^2}$$

$$h_f = \frac{4 \times 4 f l Q^2}{\pi^2 \times 2g D^5} = \frac{f l Q^2}{\left(\frac{\pi^2 2g}{16}\right) D^5} \Rightarrow h_f = \frac{f l Q^2}{12 D^5}$$

#### 2. Chezy's formula

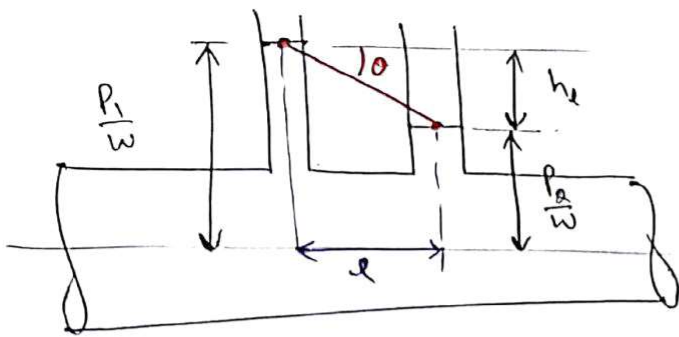
$$v = C \sqrt{mi}$$

C = Chezy's constant

v = Average velocity

m = hydraulic mean depth =  $\frac{\text{Area}}{\text{wetted perimeter}} = \frac{\frac{\pi}{4} D^2}{\pi D} = \frac{D}{4}$

i = hydraulic slope



$$l = \tan \theta = \frac{h_e}{V}$$

$$V = C \sqrt{m i}$$

$$V = C \sqrt{\frac{D}{4} \times \frac{h_e}{l}}$$

$$h_e = \frac{4 l V^2}{C^2 D}$$

$$\Leftrightarrow \frac{V^2}{C^2} = \frac{D h_e}{4 l}$$

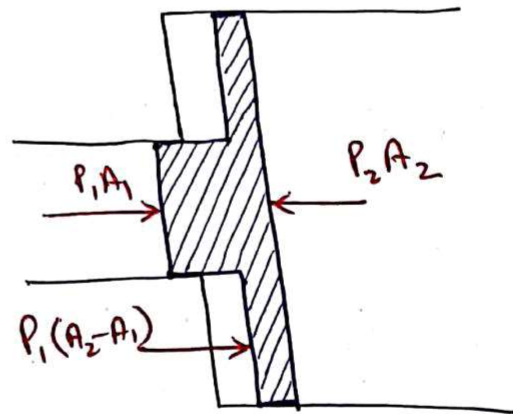
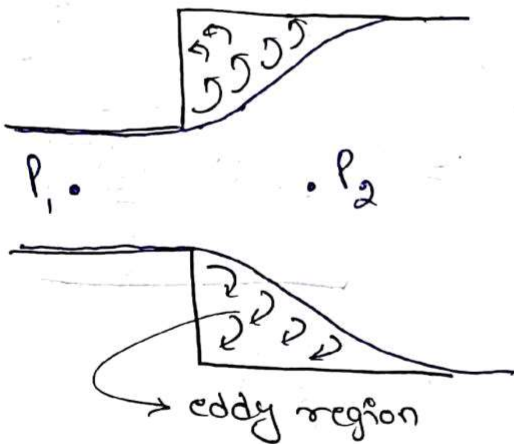
Comparison b/w C & f

$$\frac{4 l V^2}{C^2 D} = \frac{f l V^2}{2 g D}$$

$$\Rightarrow C = \sqrt{\frac{8 g}{f}} \Rightarrow \frac{\sqrt{3}}{5} \Rightarrow L^{1/2} T^{-1}$$

Minor losses [it carries 5% → generally neglected]

1. Sudden expansion loss



Assumptions : 1) Steady

2) The pressure in eddy region is taken as upstream pressure (i.e.  $P_1$ )

Continuity eqn

$$A_1 V_1 = A_2 V_2 = Q$$

$$V_2 = \frac{Q}{A_2}$$

## Momentum integral equation

$$\Sigma F = \rho Q (v - u)$$

$$P_1 A_1 + P_2 A_2 - P_1 A_1 - P_2 A_2 = \rho Q (v - u)$$

$$(P_1 - P_2) A_2 = \rho Q (v_2 - v_1)$$

$$(P_1 - P_2) = \rho v_2 (v_2 - v_1)$$

$$\frac{P_1 - P_2}{\rho g} = \frac{v_2^2 - v_1 v_2}{g} \quad \text{--- (1)}$$

B-eau<sup>n</sup> b/w (1) & (2)

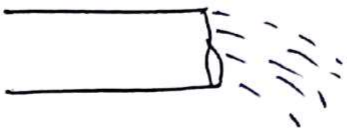
$$\frac{P_1}{\omega} + \frac{v_1^2}{2g} = \frac{P_2}{\omega} + \frac{v_2^2}{2g} + h_e$$

$$(h_e)_{S.E.} = \frac{P_1 - P_2}{\omega} + \frac{v_1^2 - v_2^2}{2g} = \frac{v_2^2 - v_1 v_2}{g} + \frac{v_1^2 - v_2^2}{2g}$$

$$(h_e)_{S.E.} = \frac{2v_2^2 - 2v_1 v_2 + v_1^2 - v_2^2}{2g} \Rightarrow \boxed{h_e = \frac{(v_1 - v_2)^2}{2g}}$$

$$h_e = \frac{v_1^2}{2g} \left(1 - \frac{v_2}{v_1}\right)^2 = \left(\frac{v_1}{2g}\right)^2 \times \left(1 - \frac{A_1}{A_2}\right)^2$$

## 2. Exit losses

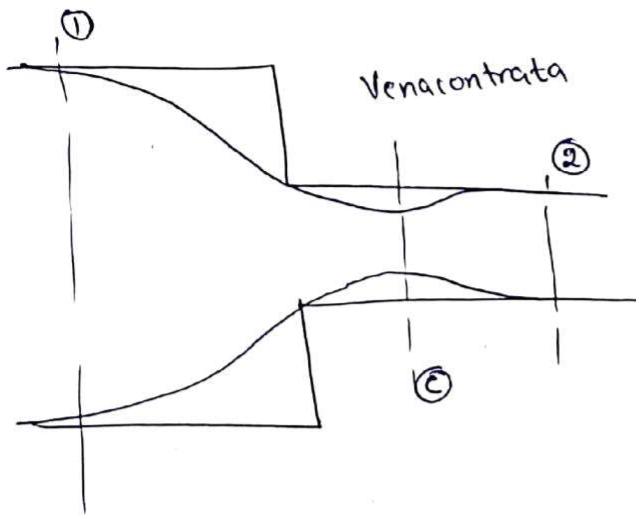


On the above eqn<sup>n</sup>  $A_2 = \infty$

$$h_e = \frac{v_1^2}{2g} \times \left(1 - \frac{A_1}{\infty}\right)^2$$

$$\Rightarrow \boxed{h_e = \frac{v_1^2}{2g}}$$

### 3. Sudden contraction losses



1 to 2

losses are neglected as area decreases Boundary layer region is small so losses are negligible

$$(h_e)_{s.c.} = \frac{(V_1 - V_2)^2}{2g} = \frac{V_2^2}{2g} \left[ \frac{V_1}{V_2} - 1 \right]^2$$

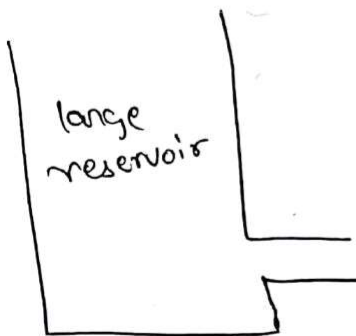
$$A_1 V_1 = A_2 V_2$$

$$\frac{V_1}{V_2} = \frac{A_2}{A_1} = \frac{1}{C_c} \Rightarrow$$

$$(h_e)_{s.c.} = \left( \frac{1}{C_c} - 1 \right)^2 \frac{V_2^2}{2g}$$

$$(h_e)_{s.c.} = 0.5 \frac{V_2^2}{2g}$$

### 4. Entry losses



$$(h_e)_{\text{entry}} = 0.5 \frac{V^2}{2g}$$

### 5. Bend losses

$$(h_e)_{\text{Bend}} = K \frac{V^2}{2g}$$

$K$  = Bending coefficient

Angle of Bend  $\theta \uparrow \Rightarrow K \uparrow$

Radius of curvature  $\uparrow \Rightarrow K \downarrow$

Radius of Pipe  $\uparrow \Rightarrow K \downarrow$

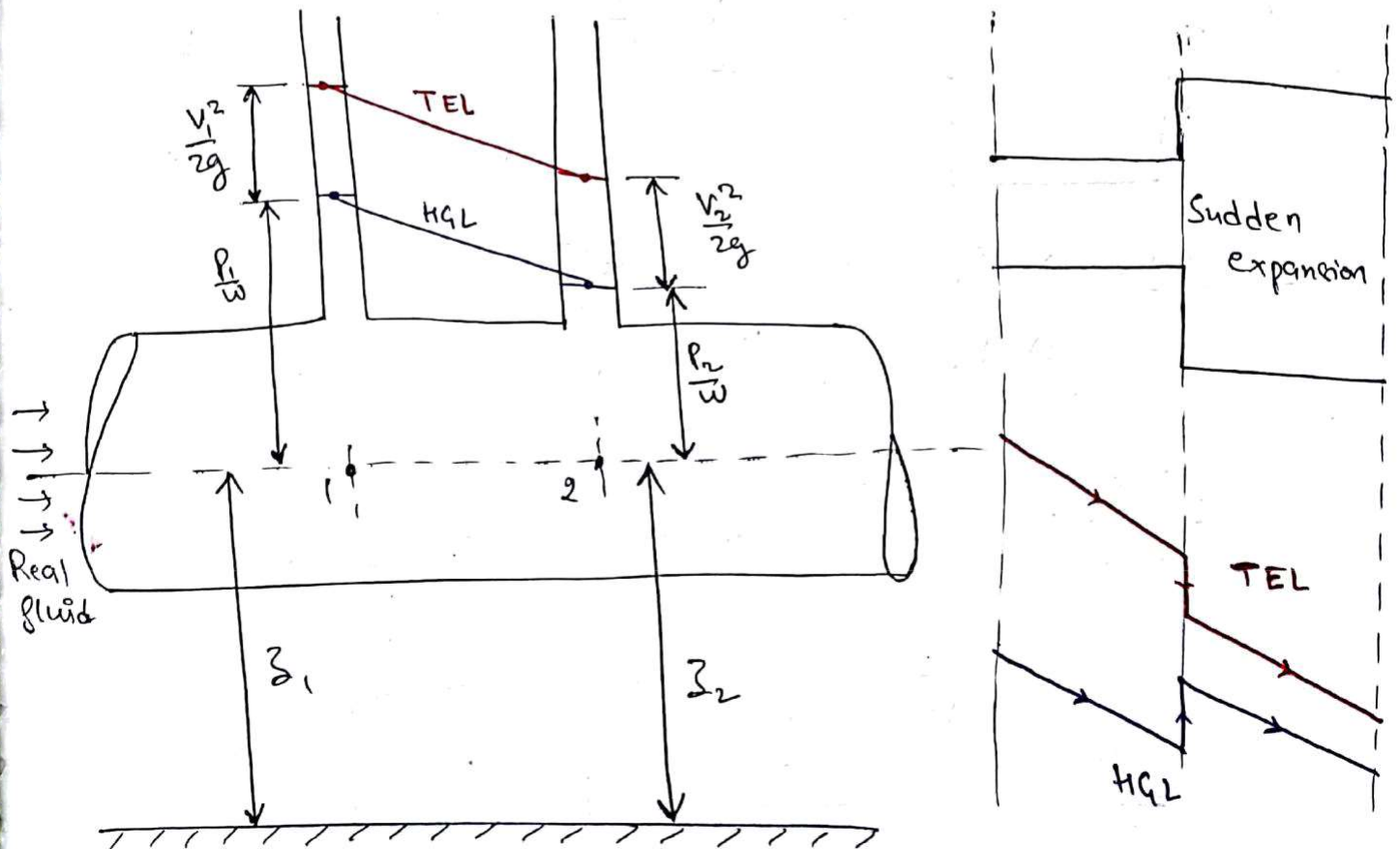


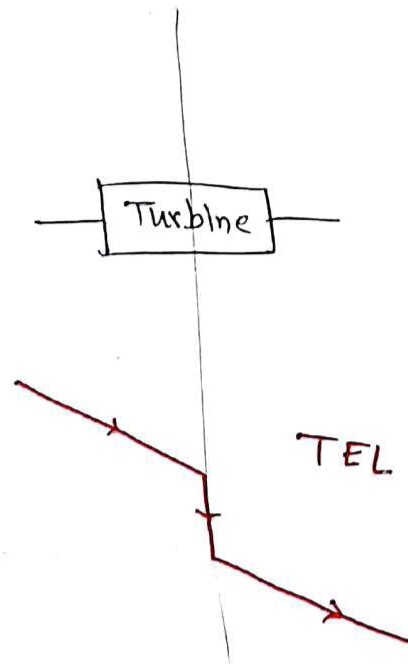
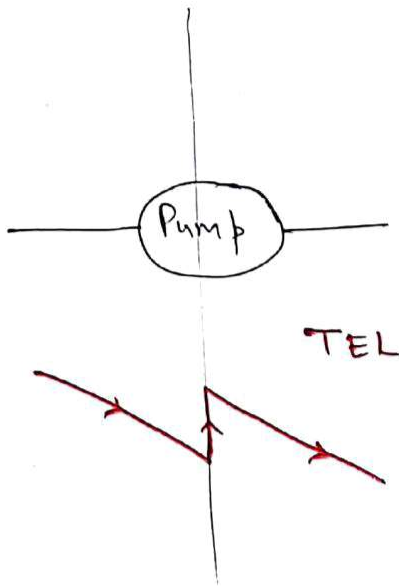
# Hydraulic gradient line & Total Energy line

(HGL)

(TEL)

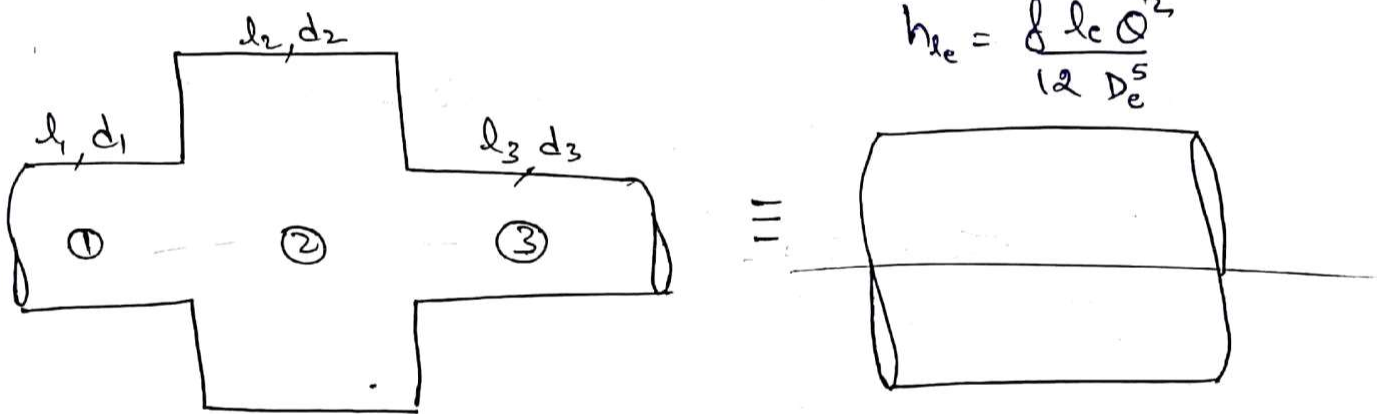
- Hydraulic Gradient line is defined as the line formed by joining the piezometric heads at various points in a fluid flow.  $\left[ \frac{p}{\rho} + z \right]$
- TEL is defined as the line formed by joining the total energies  $\left[ \frac{p}{\rho} + \frac{v^2}{2g} + z \right]$  at various points in a fluid flow.
- HGL may rise or fall ~~up~~ whereas total energy line always falls.
- TEL can only rise when energy is provided to the fluid from outside
- For an ideal fluid HGL and TEL are straight, parallel and horizontal.





## Pipes in Series & Pipes in parallel

### 1) Pipes in Series



conditions: 1)  $Q_1 = Q_2 = Q_3 = \dots = Q_n = Q = \text{const.}$

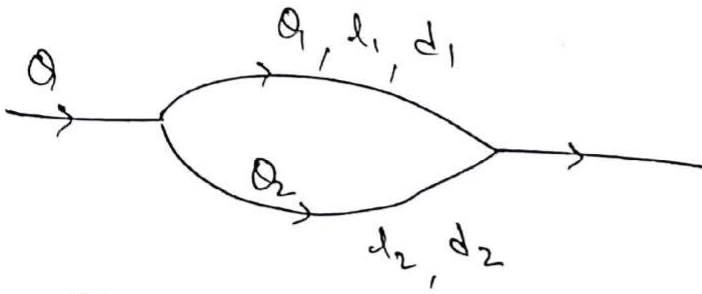
2)  $h_L = h_{L1} + h_{L2} + h_{L3} + \dots + h_{Ln}$

$$\frac{f l_e Q^2}{12 D_e^5} = \frac{f l_1 Q^2}{12 D_1^5} + \frac{f l_2 Q^2}{12 D_2^5} + \frac{f l_3 Q^2}{12 D_3^5} + \dots$$

$$\boxed{\frac{l_e}{D_e^5} = \frac{l_1}{D_1^5} + \frac{l_2}{D_2^5} + \frac{l_3}{D_3^5} + \dots}$$

Dupuits  
Formula

## 2. Pipes in Parallel



Condition 1)  $Q = Q_1 + Q_2 + Q_3 + \dots + Q_n$

Condition 2)  $h_{f1} = h_{f2} = h_{f3} = \dots = h_{fn}$

for "n" pipes  $\rightarrow$  Discharge through 1 pipe =  $\frac{Q}{n}$

$h_{fe} = (h_f)_{\text{through any pipe}}$

$$\frac{f l_e Q^2}{12 D_e^5} = \frac{8 l Q^2}{n^2 12 D^5}$$

$\Rightarrow$

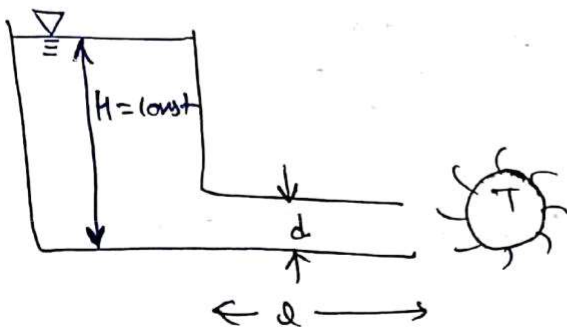
$$\boxed{\frac{l_e}{D_e^5} = \frac{l}{n^2 D^5}}$$

when all pipes are similar

$$\boxed{\sqrt{\frac{D_e^5}{l_e}} = \sqrt{\frac{D_1^5}{l_1}} + \sqrt{\frac{D_2^5}{l_2}} + \sqrt{\frac{D_3^5}{l_3}} + \dots}$$

when pipes are diff. &  $h_e$  is same.

## Power transmission through pipes



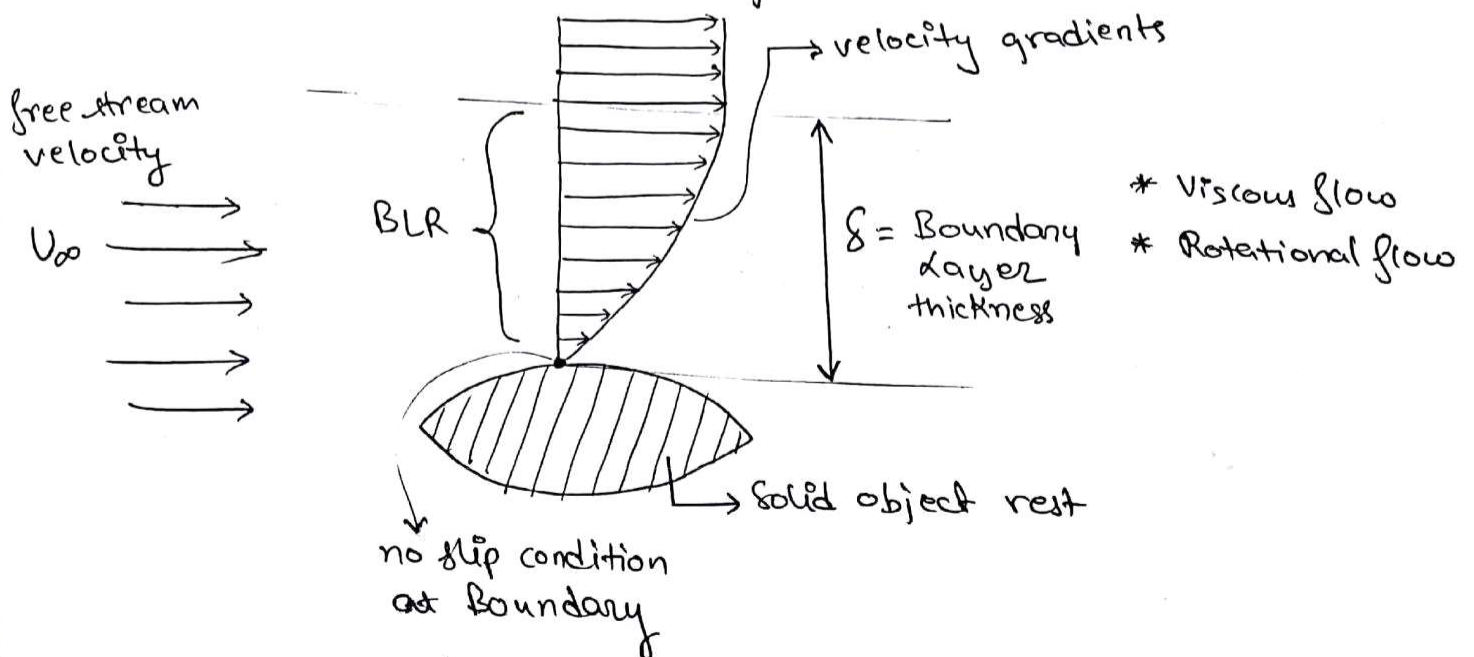
$$(Power)_{th.} = \rho g Q H$$

$$(Power)_{act.} = \rho g Q (H - h_f)$$

$$\boxed{\eta_{\text{Power trans.}} = \frac{P_{act.}}{P_{th.}} = \frac{H - h_f}{H}}$$

BOUNDARY LAYER THEORY (BLT)

- Boundary layer theory was invented by/developed by Prandtl in the year 1904. When a real fluid flows past a solid object, the velocity of the fluid on the surface of the object will be same as that of the surface.
- If the object is at rest, the fluid will also have zero velocity bcz of no slip condition at the boundary.
- Away from the boundary the velocity increases gradually and reaches free stream velocity at some distance and hence there is a small region close the boundary where velocity gradients exist. And this region is known as boundary layer region. (BLR)
- The flow in BLR is viscous and rotational.
- Outside the boundary layer region the flow is non viscous and hence the bernoulli's eqn is applicable  
\* non viscous region outside BLR



Q7 Momentum integral equation

$$\Sigma F = \rho \int v^2$$

$$\Sigma F = 10^{-6} \text{ m} \times a \text{ m}^2/\text{s}$$

$$K = 0 \text{ m} \times \frac{\text{m}}{\text{s}^2} = 4$$

$$\tau_0 = \rho (v-u)^2$$

$$\frac{K}{\rho} = \rho A v (v-u) \dots$$

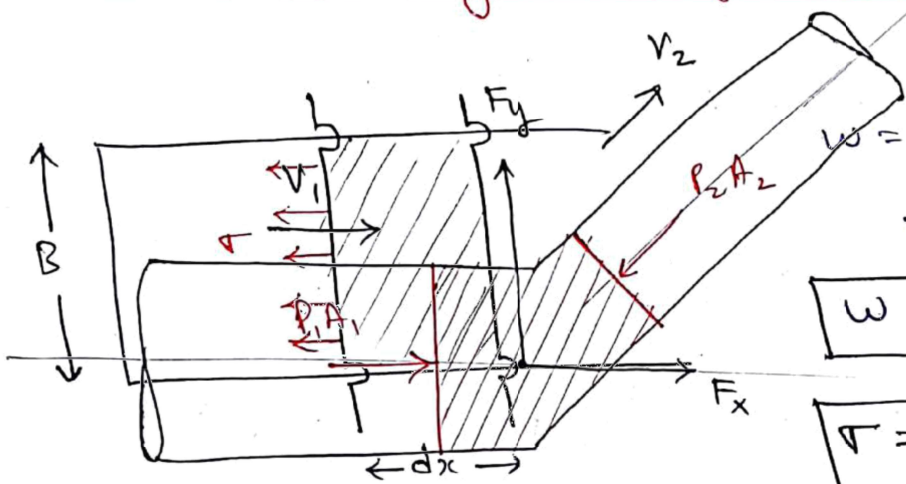
$$\tau_0 = \rho \int v^2$$

$$v^* = \sqrt{\frac{\tau_0}{\rho}}$$

$$\Sigma F = \rho \int (v-u)^2 = 11.6 \dots$$

Forces on pipe bend

Work done in increasing the surface Area



$W = \text{Force} \times \text{Displacement}$   
 $= \sigma B dx$

$$W = \sigma dA$$

$$\sigma = \frac{W}{dA}$$

Note: Surface tension is also defined as the work done per unit change in surface area and this work provided from outside is stored in the fluid film in the form of energy.

As surface tension always tries to minimise the surface area therefore for increasing the surface area work needs to be done from outside.

$$P_1 A_1 - P_2 A_2 \cos \theta + F_x = \rho \int (v_2 \cos \theta - v_1)$$

$$-P_2 A_2 \sin \theta + F_y = \rho \int (v_2 \sin \theta - 0)$$